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REPORT No. 488

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HEAT TRANSFER FROM FINNED METAL CYLINDERS IN AN AIR STREAM

By ARNOLD E. BIERMANN and BENJAMIN PINKEL



1934

U.S. 7

AERONAUTIC SYMBOLS

1. FUNDAMENTAL AND DERIVED UNITS

	Symbol	Metric		English	
		Unit	Abbrevia- tion	Unit	Abbrevia- tion
Length-----	l	meter-----	m	foot (or mile)-----	ft. (or mi.)
Time-----	t	second-----	s	second (or hour)-----	sec. (or hr.)
Force-----	F	weight of 1 kilogram-----	kg	weight of 1 pound-----	lb.
Power-----	P	horsepower (metric)-----		horsepower-----	hp.
Speed-----	V	{kilometers per hour----- meters per second-----	{k.p.h. m.p.s.	{miles per hour----- feet per second-----	{m.p.h. f.p.s.

2. GENERAL SYMBOLS

W ,	Weight = mg	ν ,	Kinematic viscosity
g ,	Standard acceleration of gravity = 9.80665 m/s ² or 32.1740 ft./sec. ²	ρ ,	Density (mass per unit volume)
m ,	Mass = $\frac{W}{g}$		Standard density of dry air, 0.12497 kg-m ⁻⁴ -s ² at 15° C. and 760 mm; or 0.002378 lb.-ft. ⁻⁴ sec. ²
I ,	Moment of inertia = mk^2 . (Indicate axis of radius of gyration k by proper subscript.)		Specific weight of "standard" air, 1.2255 kg/m ³ or 0.07651 lb./cu.ft.
μ ,	Coefficient of viscosity		

3. AERODYNAMIC SYMBOLS

S ,	Area	i_w ,	Angle of setting of wings (relative to thrust line)
S_w ,	Area of wing	i_s ,	Angle of stabilizer setting (relative to thrust line)
G ,	Gap	Q ,	Resultant moment
b ,	Span	Ω ,	Resultant angular velocity
c ,	Chord	$\frac{Vl}{\mu}$,	Reynolds Number, where l is a linear dimension (e.g., for a model airfoil 3 in. chord, 100 m.p.h. normal pressure at 15° C., the cor- responding number is 234,000; or for a model of 10 cm chord, 40 m.p.s. the corresponding number is 274,000)
b^2 ,	Aspect ratio	C_p ,	Center-of-pressure coefficient (ratio of distance of $c.p.$ from leading edge to chord length)
\bar{S} ,	True air speed	α ,	Angle of attack
V ,		ϵ ,	Angle of downwash
q ,	Dynamic pressure = $\frac{1}{2}\rho V^2$	α_∞ ,	Angle of attack, infinite aspect ratio
L ,	Lift, absolute coefficient $C_L = \frac{L}{qS}$	α_i ,	Angle of attack, induced
D ,	Drag, absolute coefficient $C_D = \frac{D}{qS}$	α_a ,	Angle of attack, absolute (measured from zero- lift position)
D_o ,	Profile drag, absolute coefficient $C_{D_o} = \frac{D_o}{qS}$	γ ,	Flight-path angle
D_i ,	Induced drag, absolute coefficient $C_{D_i} = \frac{D_i}{qS}$		
D_p ,	Parasite drag, absolute coefficient $C_{D_p} = \frac{D_p}{qS}$		
C ,	Cross-wind force, absolute coefficient $C_C = \frac{C}{qS}$		
R ,	Resultant force		

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IN AN AIR STREAM**

By **ARNOLD E. BIERMANN** and **BENJAMIN PINKEL**
Langley Memorial Aeronautical Laboratory

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SUMMARY

This report presents the results of tests made by the National Advisory Committee for Aeronautics to supply design information for the construction of metal fins for the cooling of heated cylindrical surfaces by an air stream.

Heat-transfer coefficients were obtained over a range of air speeds from 30 to 150 miles per hour from tests in a wind tunnel of a series of electrically heated finned steel cylinders, which covered a range of fin pitches from 0.10 to 0.60 inch, average fin thicknesses from 0.04 to 0.27 inch, and fin widths from 0.37 to 1.47 inches. Tests were also conducted on a smooth steel cylinder without fins.

The quantity of heat dissipated, calculated from a theoretical equation using the surface heat-transfer coefficients found from the experiments, was compared with the experimentally determined quantity of heat dissipated. The agreement was found sufficiently good to justify the use of the theoretical formula for calculating the quantity of heat dissipated from the finned metal cylinders.

A method is presented for determining fin dimensions for a maximum heat transfer with the expenditure of a given amount of material for a variety of conditions of air flow and metals.

INTRODUCTION

The cooling of hot bodies by means of metal fins exposed to an air stream can be treated as two related problems, one involving the convection of heat from the fin surfaces by the air stream and the other involving the conduction of heat through the fins to the fin surfaces. The rate at which heat is conveyed from a surface by an air stream is usually expressed as a surface heat-transfer coefficient (q). Theoretical methods have been developed for calculating the surface heat-transfer coefficients for simple surfaces over which the air flow is known (reference 1). For complex bodies, such as finned cylinders, the velocity field over the surface of the fins is extremely complicated, especially in the region at the rear half of the cylinder where the flow is vortical. For such bodies experimental methods must be used at present for determining the surface heat-transfer coefficient.

The second problem—the conduction of the heat through the fins—is simpler and the associated differ-

ential equation permits of solution in terms of functions of the fin dimensions and the surface heat-transfer coefficient. Solutions have been given by Harper and Brown (reference 2) and Schmidt (reference 3) for various types of fins.

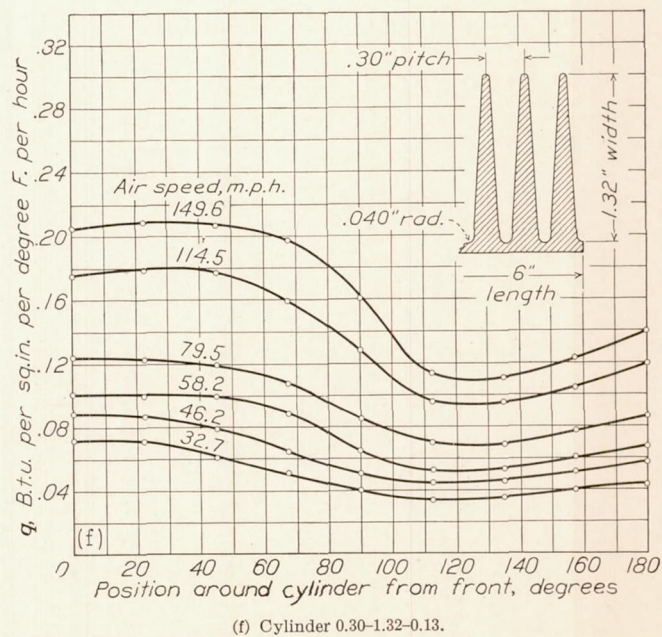
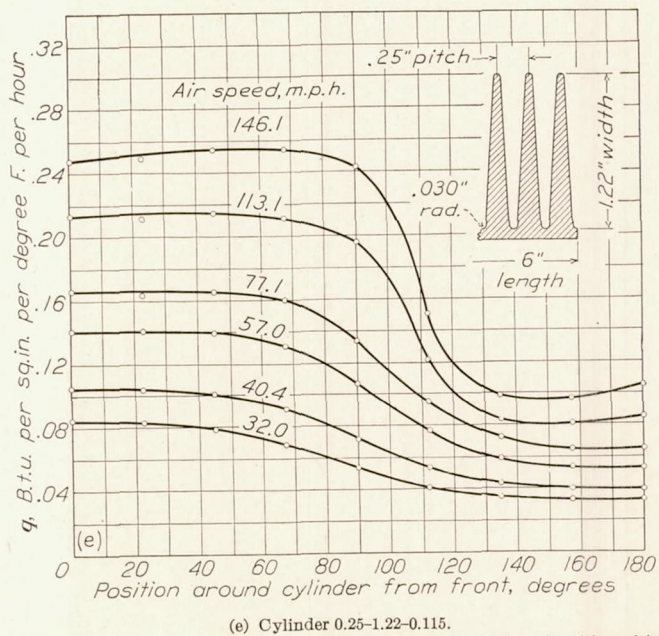
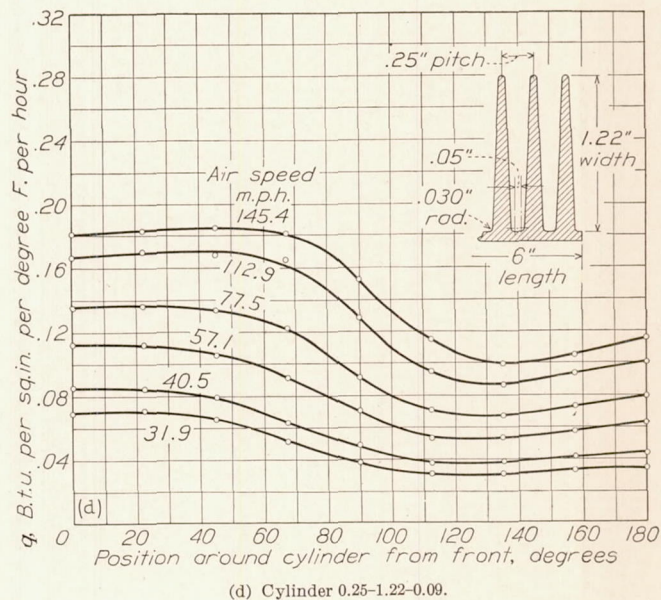
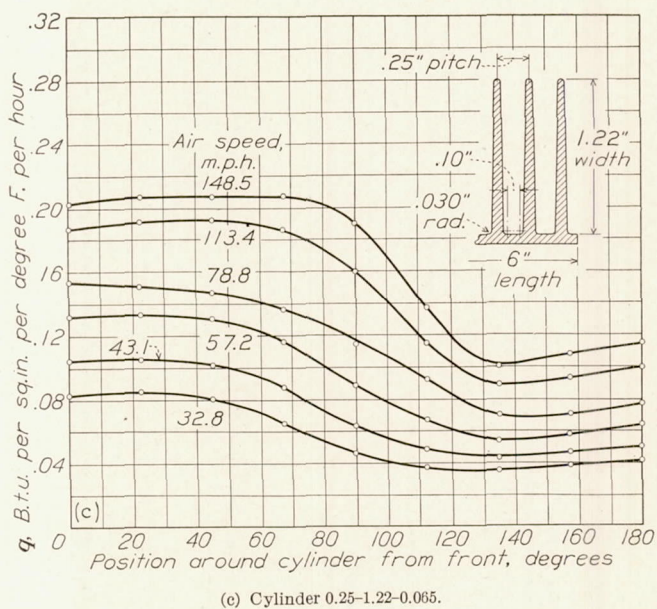
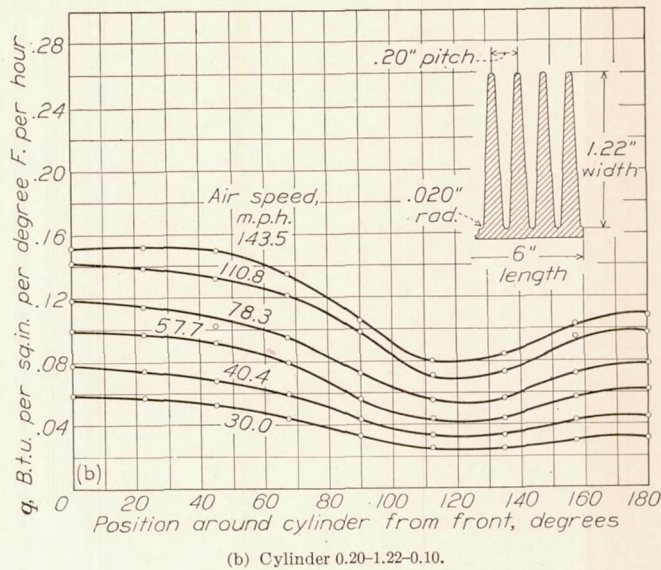
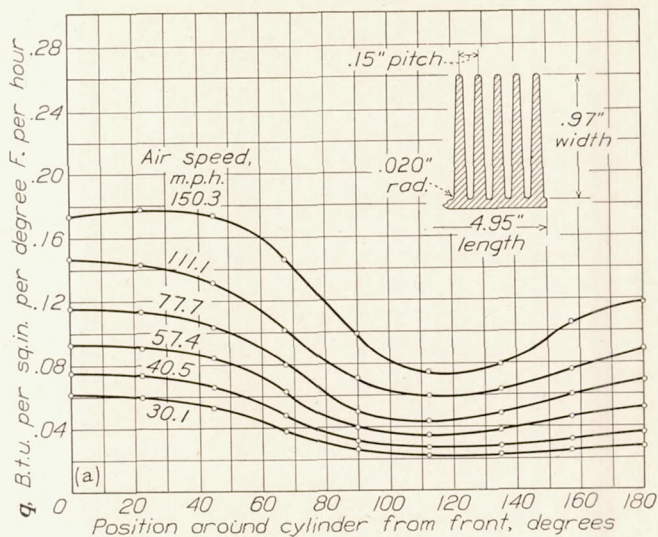
The present paper reports part of a general research on the cooling of finned cylinders. It covers an investigation conducted during 1932 and 1933 by the National Advisory Committee for Aeronautics to supply information for the design of fins for air-cooled engine cylinders operating in a free air stream. In this respect it was considered desirable to:

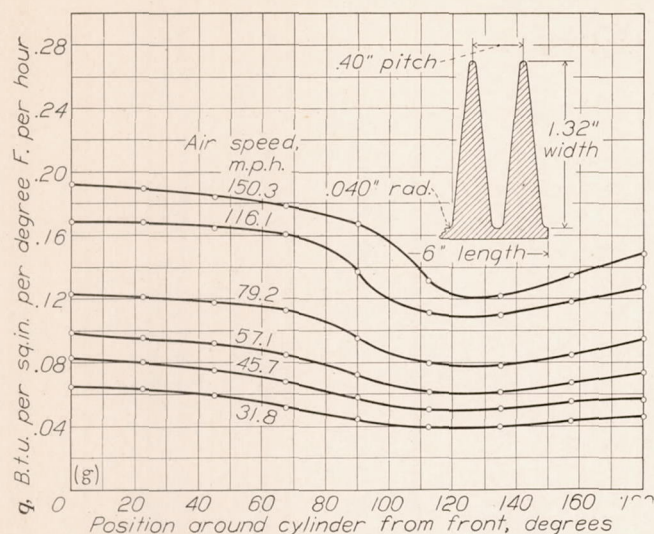
1. Compare the relative advantages of different fin shapes.
2. Obtain experimental values of the surface heat-transfer coefficient for a variety of conditions of fin shape, width, thickness, and spacing for a wide range of air speeds.
3. Derive a method of calculating the quantity of heat dissipated from finned cylinders.
4. Develop a method for determining the fins having a maximum heat transfer for any given expenditure of material.

APPARATUS

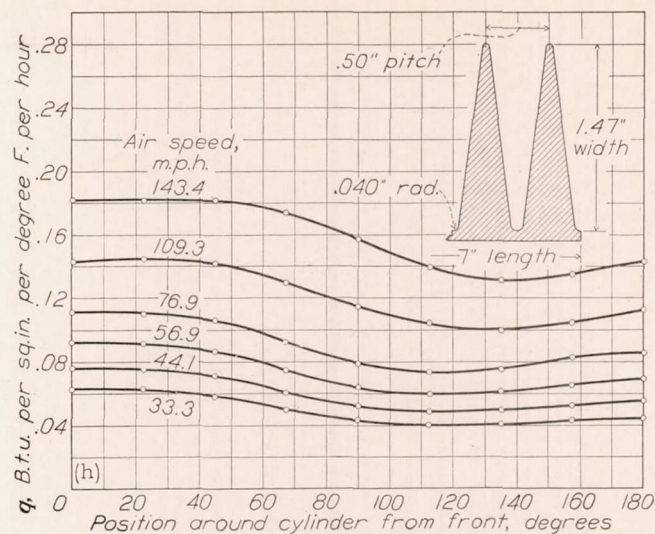
A group of 18 steel cylinders having inside diameters of 4.5 inches and wall thicknesses of 0.08 inch were selected for testing. Each cylinder was machined from a forging of S.A.E. 1050 steel, heat-treated to give a Brinell hardness of approximately 200. This construction is commonly used for aircraft-engine cylinder barrels. The natural, semipolished surface was maintained during the tests.

For convenience in referring to the finned cylinders a descriptive nomenclature based on the fin pitch, width, and thickness has been devised. For example, the designation "0.25-0.67-0.04" indicates a finned cylinder having a fin pitch of 0.25 inch, a fin width of 0.67 inch, and an average fin thickness of 0.04 inch. Sketches showing cross sections of the fin constructions of the various cylinders are presented in figure 1. All the fins having a thickness of 0.04 inch were rectangular in cross section; the other fins were tapered. The

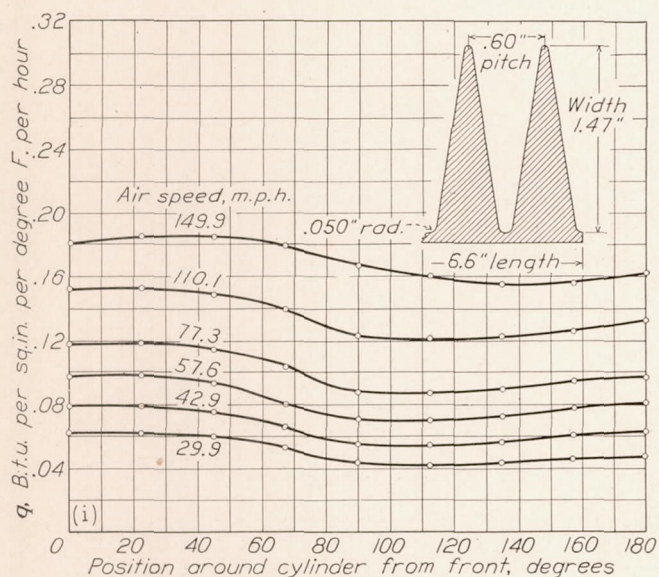
FIGURE 1.—Variation of q with position around the cylinder at different air speeds.



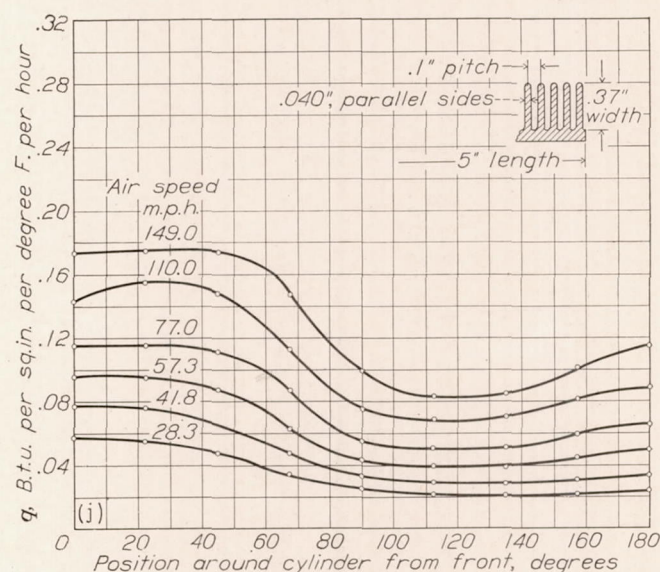
(g) Cylinder 0.40-1.32-0.18.



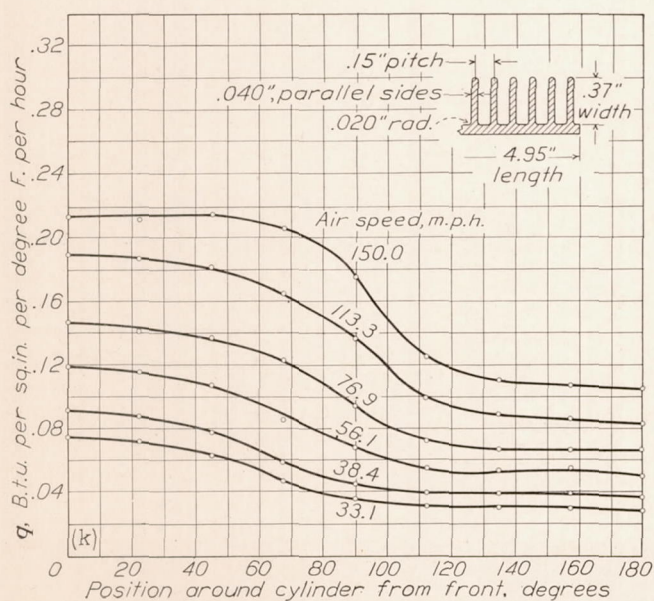
(h) Cylinder 0.50-1.47-0.23.



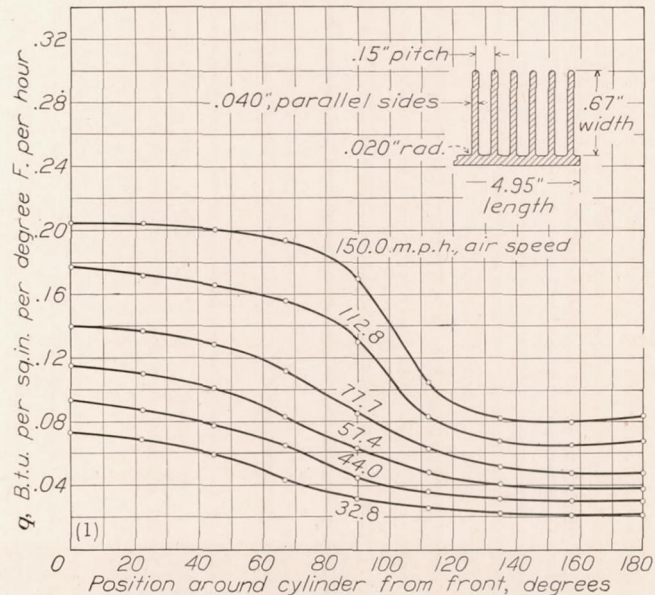
(i) Cylinder 0.60-1.47-0.27.



(j) Cylinder 0.10-0.37-0.04.

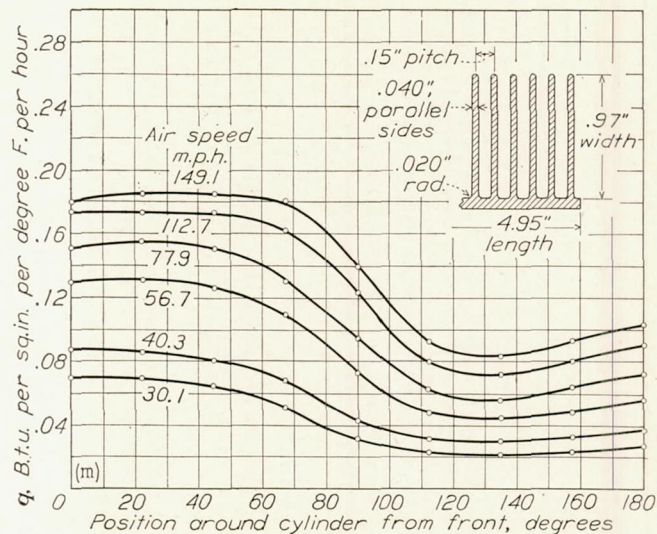


(k) Cylinder 0.15-0.37-0.04.

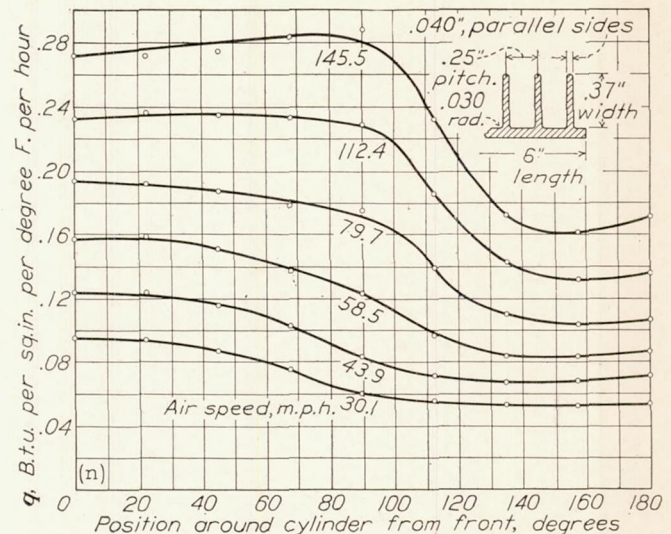


(l) Cylinder 0.15-0.67-0.04.

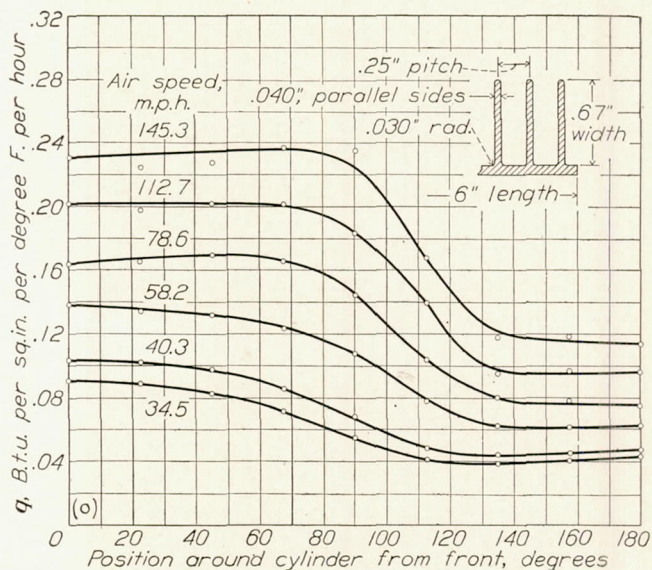
FIGURE 1.—Variation of q with position around the cylinder at different air speeds.—Continued.



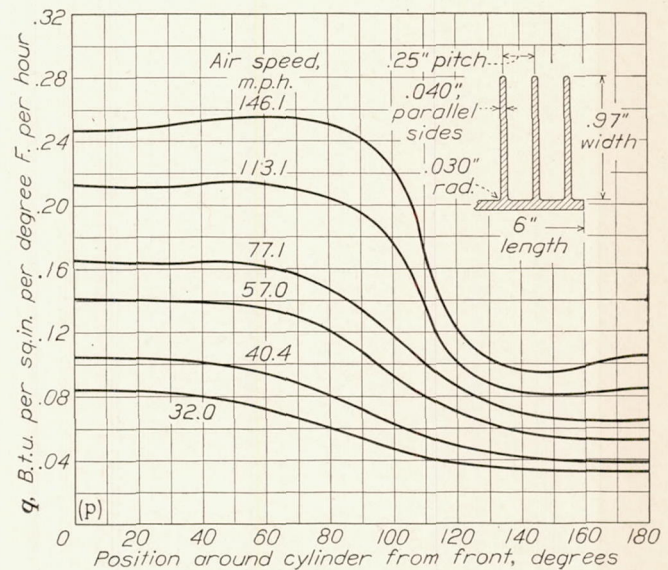
(m) Cylinder 0.15-0.97-0.04.



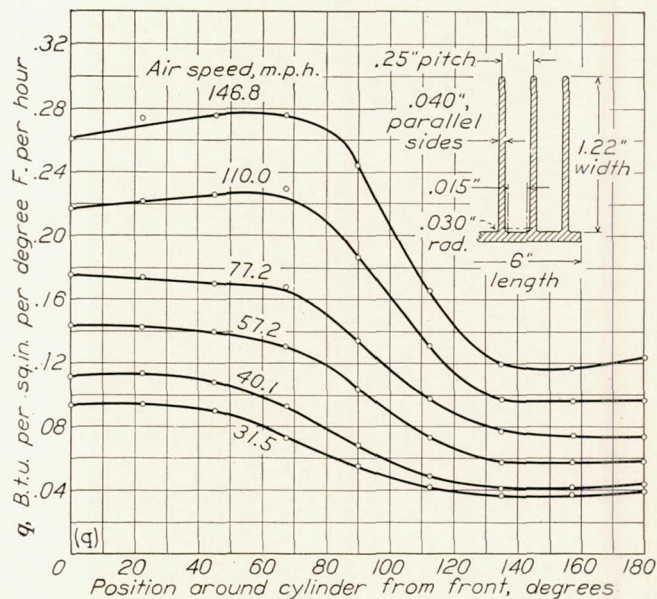
(n) Cylinder 0.25-0.37-0.04.



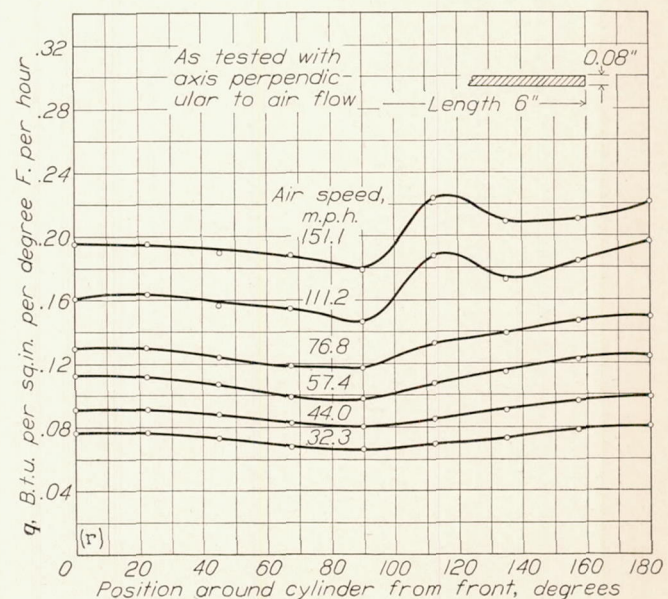
(o) Cylinder 0.25-0.67-0.04.



(p) Cylinder 0.25-0.97-0.04.



(q) Cylinder 0.25-1.22-0.04.



(r) Smooth cylinder.

FIGURE 1.—Variation of q with position around the cylinder at different air speeds.—Continued.

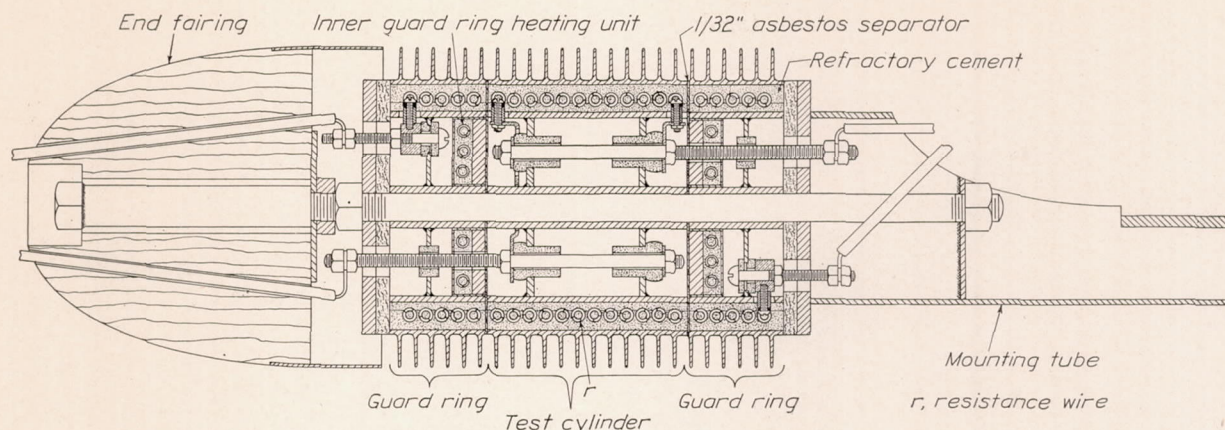


FIGURE 2.—Details of construction of test unit.

radius of all the fin tips was 0.02 inch. The various cylinders tested are grouped in the following table:

With wedge-shaped fins	With rectangular fins
0.15-0.97-0.075	0.10-0.37-0.04
.20-1.22-.10	.15-.37-.04
.25-1.22-.065	.15-.67-.04
.25-1.22-.09	.15-.97-.04
.25-1.22-.115	.25-.37-.04
.30-1.32-.13	.25-.67-.04
.40-1.32-.18	.25-.97-.04
.50-1.47-.23	.25-1.22-.04
.60-1.47-.27	Smooth cylinder

The cylinders were electrically heated and tested with guard rings as in previous tests (reference 4). The guard rings served to eliminate heat flow through the ends and also to obtain two-dimensional air flow over the cylinders. Figures 2 and 3 show the construction of the heating elements of a test cylinder and the guard rings and figure 4 shows guard rings assembled with a test cylinder.

The heat input to the test cylinder and to each of the guard rings was regulated by means of separate oil-submerged wire rheostats of the continuously variable type. A wiring diagram for the heating units, rheostats, and instruments is shown in figure 5.

Surface temperatures were measured by means of 24 iron-constantan thermocouples, used in conjunction with a direct-reading pyrometer and a selector switch. The thermocouple junctions were constructed by spot-welding 0.013-inch iron and constantan wires to the cylinder surface.

Of the 24 thermocouples used, 9 were located on the outer cylinder wall between fins at intervals of $22\frac{1}{2}^\circ$ around one-half of the cylinder from front to rear, and 9 were similarly located on the fin tips. Additional thermocouples were placed at intermediate positions across the fin width at the front, side, and rear of the cylinder. Temperatures on the smooth cylinder tested parallel to the air stream were measured by 10 thermocouples placed along a spiral path on the outer surface extending from the front to the rear.

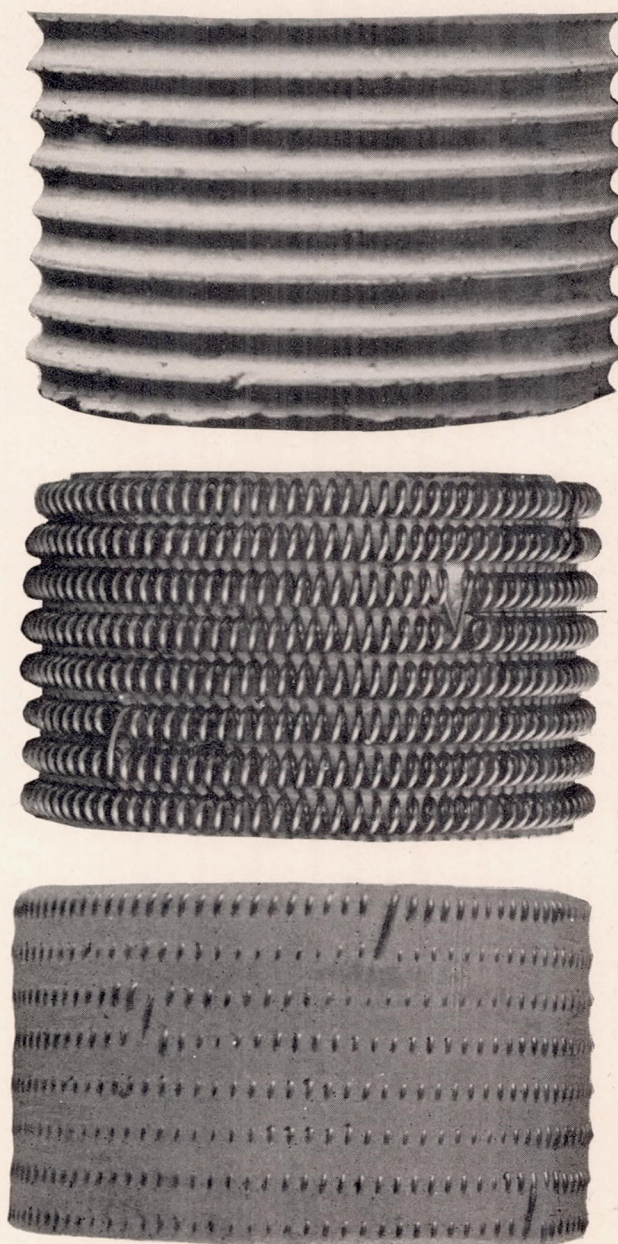


FIGURE 3.—Steps in the construction of a heating unit.

Differential thermocouples were provided for determining when the guard-ring temperatures were equal

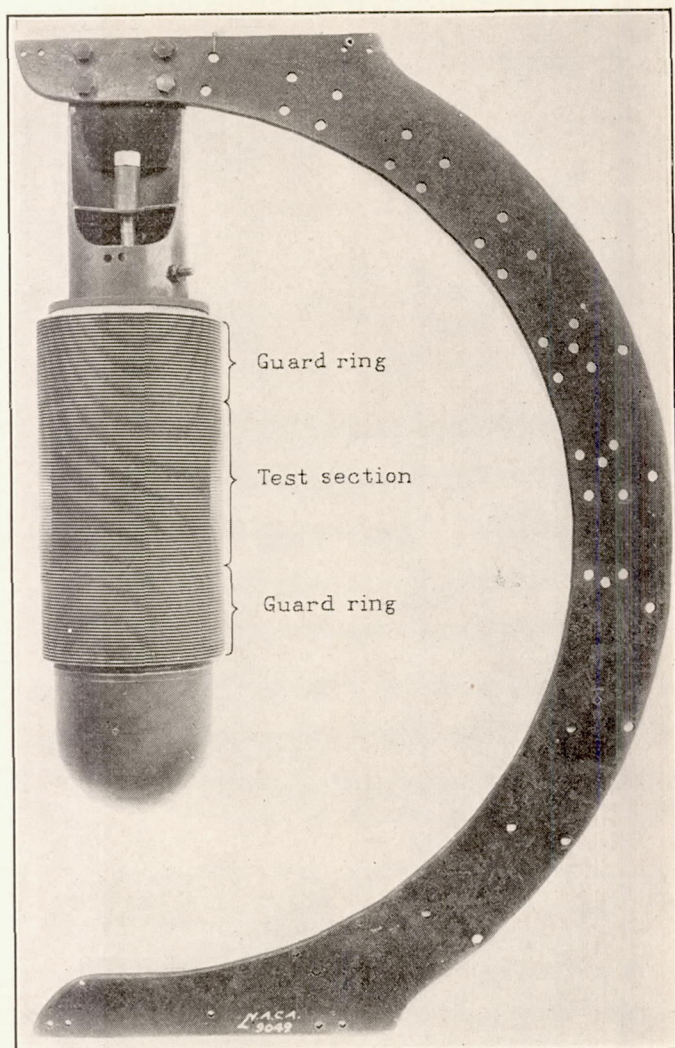


FIGURE 4.—Assembly of (0.15-0.37-0.04) test unit.

The 30-inch closed-throat wind tunnel used in these tests is shown in figure 6. (See reference 4.)

The tunnel air speed was measured by means of a pitot-static tube in conjunction with a water manometer. The pitot-static tube was located at one side of the test unit and sufficiently far ahead so that the presence of the cylinder had no measurable effect upon the reading.

PRECISION

Improvements in the guard-ring system over that used in previous tests have lowered the axial heat flow from approximately 6 percent to 1 or 2 percent. The

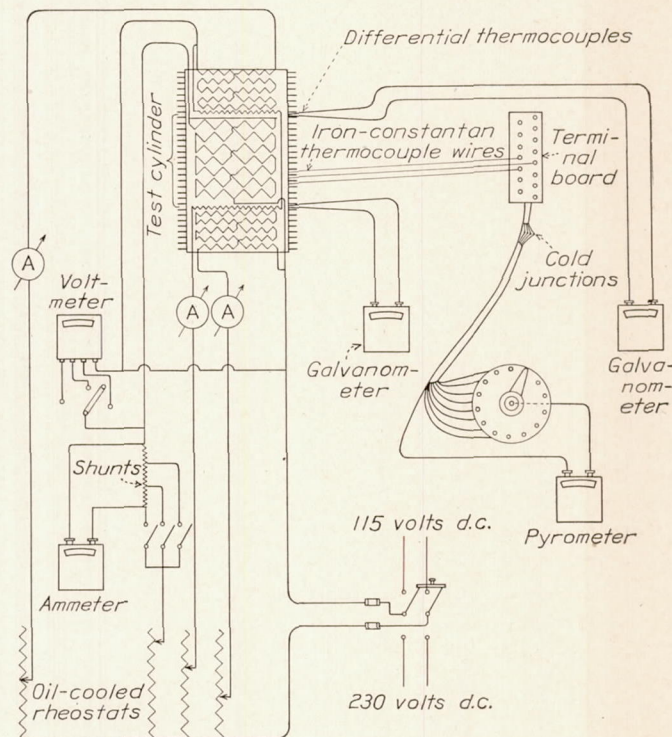


FIGURE 5.—Wiring diagram of test set-up.

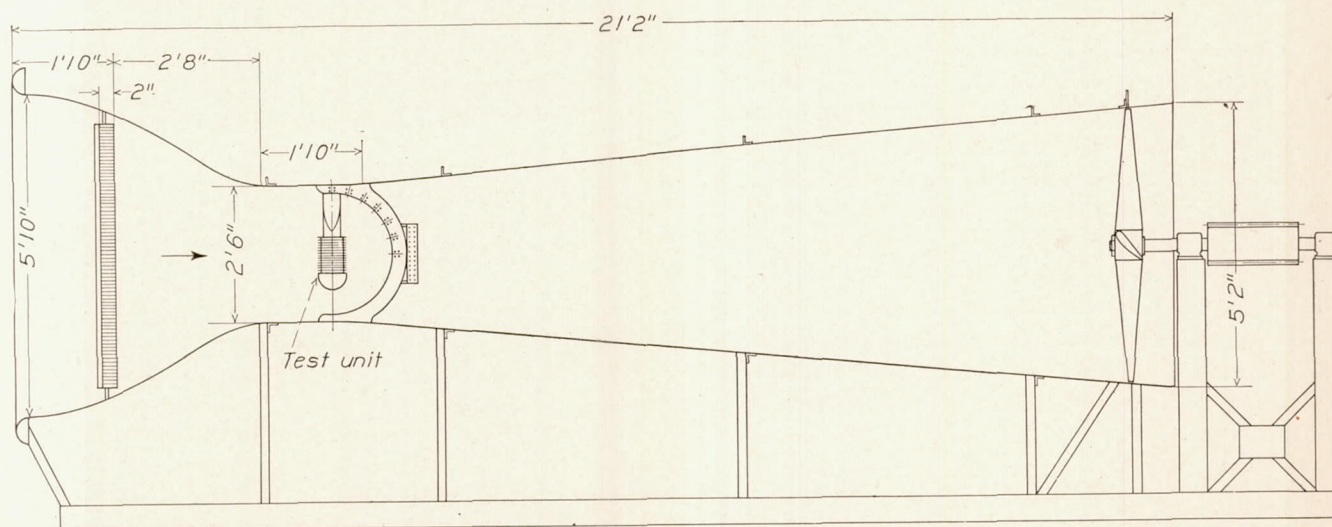


FIGURE 6.—Wind tunnel showing test unit mounted in place.

to that of the test cylinder. The junctions of these thermocouples were located at adjacent positions on the guard rings and the test cylinder.

temperature distribution along the cylinder surface parallel to the axis was, in all specimens, considerably more uniform than in previous tests.

At the beginning of this investigation a series of tests was conducted to determine the effect on thermocouple readings of wire diameters from 0.013 to 0.052 inch, of leading the wires directly away from the surface, of placing the wire next to the surface for a distance away from the junction, and of embedding the wires in a narrow groove in the surface. Tests were conducted on enameled wire both bare and silk-covered, and on enameled wire shellacked to the cylinder surface. The results indicate no appreciable difference in any of the methods for leading the wires away from the surface, provided that the wire is of less than 0.025-inch diameter. The precision of the temperature measurements is believed to be within $\pm 2^\circ \text{F}$.

The ammeter and voltmeter used in measuring the electrical input each had a precision of 0.5 percent of full-scale deflection. The maximum error at a minimum input was approximately 3 percent.

The air-speed measurements are believed to be accurate within 1 percent for speeds above 40 miles per hour, and within 3 percent below this speed.

METHODS

TESTS

Tests were conducted at approximately constant heat input for each finned cylinder and at air speeds ranging from 30 to 150 miles per hour. The smooth cylinder was tested up to 215 miles per hour with its axis parallel to the air stream. The heat input for the 18 cylinders ranged from 23.4 to 101.3 B.t.u. per square inch of outer cylinder-wall area per hour.

CALCULATIONS

Air speed.—Since the heat transfer is a function of the product of the velocity and density of the air, all air-speed readings were corrected to a common density which corresponds to a barometric pressure of 29.92 inches of Hg and an air temperature of 80°F .

Surface heat-transfer coefficients.—The average surface heat-transfer coefficient q was obtained by dividing the total heat input per hour by the product of the total cooling surface area and the average temperature of the entire cooling surface. The term "temperature" as used here and throughout this report will refer to the temperature difference between the heated surface and the cooling air.

An approximate q for individual stations around each cylinder was obtained by dividing the total heat input per square inch of total cooling surface area by the average surface temperature at each station.

RESULTS AND DISCUSSION

SHAPE OF FINNED

An extensive study of fin shapes has been made by Schmidt (reference 3) with the object of determining the profile of the lightest fin that will dissipate a

required amount of heat for a given root temperature. Such fins are shown to be characterized by zero tip thickness and by having a linear temperature variation from the root to the tip, the tip temperature being that of the free air stream. The required fin shape is a function of the surface heat-transfer coefficient and, for complex bodies such as finned cylinders over the surfaces of which the coefficient varies considerably, its exact determination is difficult. For the simple case of a constant surface heat-transfer coefficient over the entire fin the optimum profile was found to be composed of two congruent parabolas having their vertexes tangent to the median line at the fin tip. However, for the same cooling requirements the triangular fin was found to be only 4.5 percent heavier. Since the tip must have a definite thickness, which for the thin fins used in modern air-cooled engines may be almost as great as the fin root thickness, the importance of determining the exact profile is lessened. From these considerations the tapered fin appears as the practical fin profile which best combines effective cooling with ease of manufacture.

For any given condition a temperature survey along the width of the fin will show how the shape of a fin may be improved. Figure 7 shows a temperature survey taken along the fin width at the front, side, and rear of three different cylinders and shows the effect of thickening the fin root on the temperature distribution.

SURFACE HEAT-TRANSFER COEFFICIENTS

The problem of the transfer of heat from surfaces is closely related to skin friction and boundary-layer theory. The same mechanism that transmits momentum through a boundary layer also transmits heat. The heat transfer is greatest from those surfaces having a thin turbulent boundary layer of high average velocity.

For the case of potential flow about a cylinder the average velocity is highest at the cylinder surface and decreases to the velocity of the free air stream as the distance from the surface is increased. Actually, owing to viscous effects, a boundary layer is built up on the surface of the cylinder causing the flow to break away from the cylinder at about 100° from the front to form a vortical wake at the rear of the cylinder. The flow is further modified by the presence of fins over the surfaces of which individual boundary layers are built up.

Effect of fin width on q .—Although the flow is considerably different from potential flow, the average velocity over narrow fins may still be expected to be greater than over wide fins, resulting in a higher surface heat-transfer coefficient for the former case. The effect of fin width on q is shown in figure 8 as determined from tests on finned cylinders having pitches of 0.15 inch and 0.25 inch. Increase in fin width up to

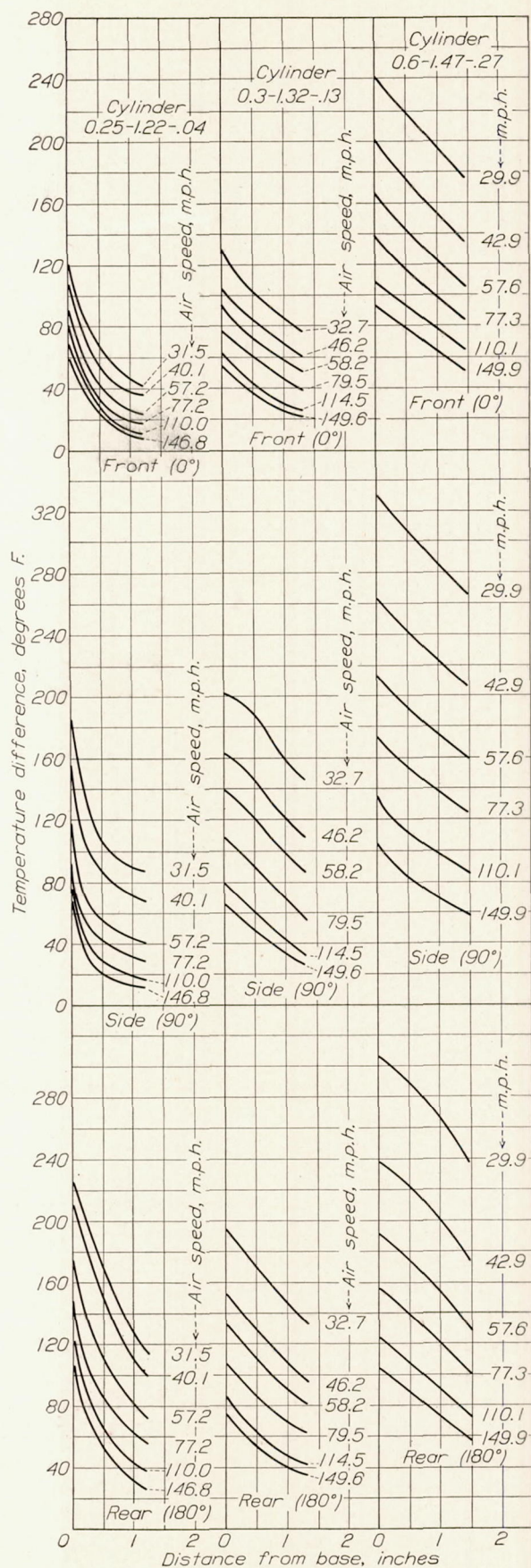


FIGURE 7.—Temperature difference along the fin width at the front, side, and rear of three cylinders tested.

about 0.40 inch results in an appreciable reduction in the value of q ; whereas for a further increase in width the variation in the value of q is much smaller. As most fins of practical interest have a width greater than 0.40 inch it may be assumed for purposes of calculation that q is independent of fin width.

Effect of fin space on q .—For closely spaced fins the mutual interference of the boundary layers of adjacent fins restricts the air flow and results in a small surface heat-transfer coefficient. As the fin spacing is increased the interference decreases and q increases, until a point is reached where the flows over adjacent fins no longer interfere and q approaches a limiting value.

Curves representing an average of all the surface heat-transfer coefficients determined in the tests are shown in figure 8 plotted against the average air space, s , between adjacent fins. These curves represent tests

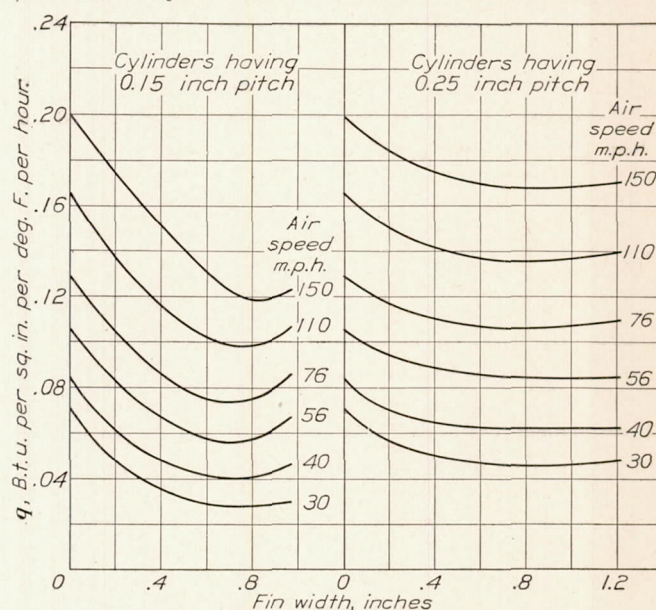


FIGURE 8.—Effect of varying the fin width on the average values of q for cylinders having fins of 0.04-inch thickness and 0.15-inch and 0.25-inch pitch.

on rectangular and tapered fins of various thicknesses, widths, and taper angles. The average deviation of the test points from the curves is less than 8 percent, showing that the most important fin dimension governing the value of q is s and that the effect of variations in the other dimensions is small. For the range of spacings tested the value of q varies with the 0.322 power of s .

Effect of fin thickness on q .—For a given value of s , variation in fin thickness does not have an appreciable effect on the air flow. Sufficient data were not collected to determine the independent effect of fin thickness. As previously mentioned, the thickness was found to be of minor importance in determining the value of q . For instance, the 0.25-1.22-0.09 and the 0.3-1.32-0.13 cylinders have approximately the same fin space, the first having a thickness of 0.09 inch and a width of 1.22 inches and the second a thickness

of 0.13 inch and a width of 1.32 inches. In spite of these differences, the value of q at 76 miles per hour for the first-mentioned finned cylinder is 0.091 and for the second is 0.087.

Variation of q with velocity.—The surface heat-transfer coefficients from figure 9 were plotted against velocity on logarithmic coordinates for a space of 0.175 inch (fig. 10). The slope of the curve shows that the value of q varies with the 0.796 power of the velocity. This exponent holds fairly well for the range of fin spacings tested. For very close spacing a transition point is indicated at low velocities. From theoretical considerations the value of q for flat plates and tubes has been shown to vary with the 0.5 power of velocity for the case of laminar flow in the boundary layer and

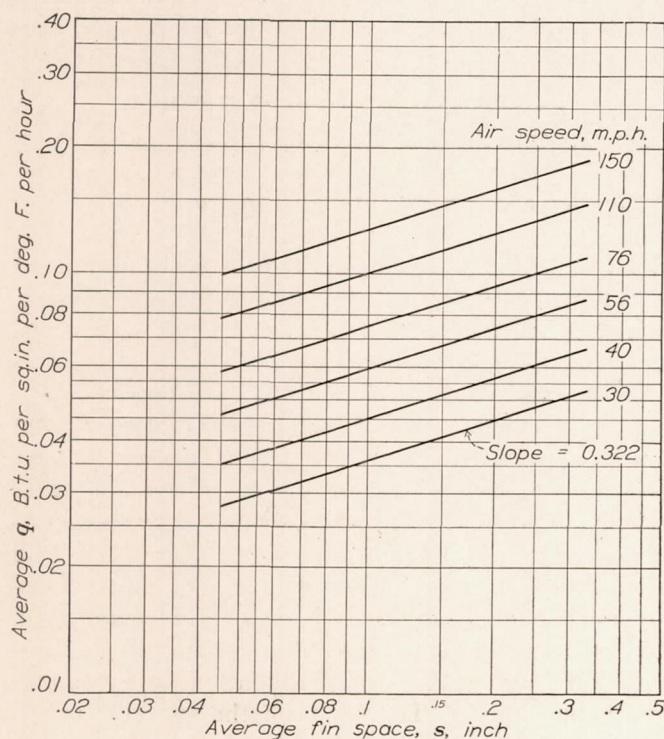


FIGURE 9.—Variation of average q with the average fin space for all cylinders tested.

with the 0.8 power of velocity for the case of turbulent flow.

The effect of velocity on q for a smooth cylinder tested with the axis both parallel and perpendicular to the air flow is also shown in figure 10. Each curve shows a transition point which, for the case of the axis perpendicular to the flow, occurs at a Reynolds Number of 109,000. Fage (reference 5) shows from tests on the drag of circular cylinders having their axes perpendicular to the air stream that a critical Reynolds Number exists between 100,000 and 500,000, the higher the initial turbulence of the air stream the lower the critical Reynolds Number.

Values of q at various points around the cylinder.—Values of q have been presented thus far in terms of the average for the entire cylinder. As the air flow varies

over different portions of the fins, the value of q necessarily changes from point to point. Values of q at individual points around the cylinder are shown in figure 1 together with a sectional view of each finned cylinder tested. These values are not the true q 's for the individual positions around the cylinder, as in their determination no account was taken of the circumferential heat flow through the fins and cylinder wall. However, for approximate calculation this uncorrected value is probably of more use than the true value, since for a given cylinder it allows the calculation of the heat dissipated from the given point on the cylinder directly from the heat-transfer equations without necessitating a determination of the circumferential flow through the metal. When this coefficient is applied to fins of

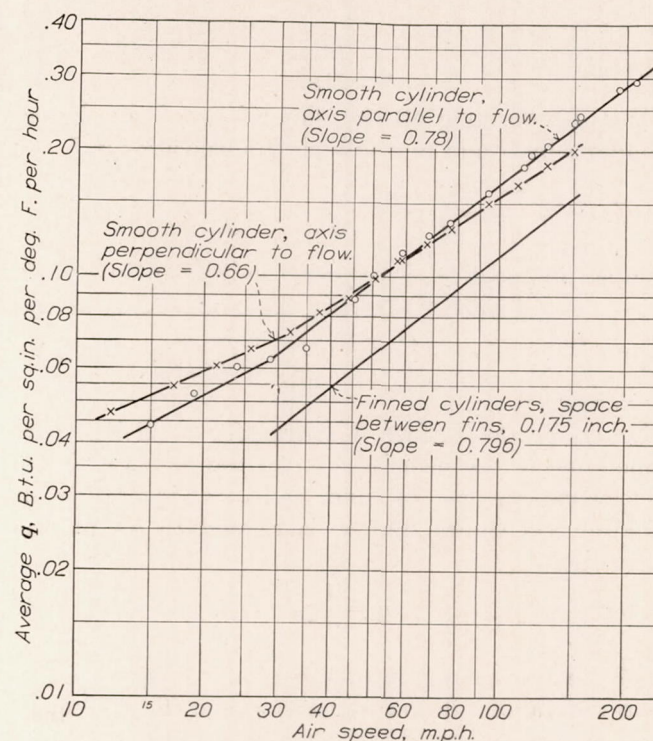


FIGURE 10.—Effect of air speed on the surface heat-transfer coefficient.

another cylinder at a similar point on the circumference, it will be in error by the difference in the circumferential heat flow for the two cylinders. The error involved, however, is sufficiently small to permit the use of the coefficient for estimating the heat dissipated from various positions on the cylinder.

Correction for cylinder diameter.—The values of q presented apply only to a cylinder diameter of 4.66 inches and an air density corresponding to sea-level conditions (29.92 inches of Hg and 80° F.). The value of q can be determined for other cylinder diameters and air densities by use of the theory of similitude.

An analysis of the theoretical equations and experimental data (references 1 and 6) on heat transfer show that the important factors affecting the value of q are the conductivity, density, viscosity, and specific heat

of the cooling air, and the various dimensions determining the geometry of the body and its position relative to the air stream. By dimensional analysis q is set up as a function of these quantities for a finned cylinder having its axis perpendicular to the air stream, and for which two-dimensional flow is assumed,

$$q = c_p \rho V f \left(\frac{\rho V D}{\mu}, \frac{k_a}{\mu c_p}, \frac{t}{D}, \frac{w}{D}, \frac{s}{D} \right) \quad (1)_f$$

using the conventional symbols defined at the end of the report.

Let D_x be the diameter of the finned cylinder x for which the value of q is required, and let t_x , w_x , and s_x be the dimensions of the fins and V_x the air speed. Let D_T be the diameter of the tested cylinders T (4.66 inches).

$$\text{Define} \quad J = \frac{D_x}{D_T} \quad (2)$$

Then by dividing all the dimensions of the finned cylinder x by J and multiplying the velocity by J ,

$$D_T = \frac{D_x}{J}; \quad t_T = \frac{t_x}{J}; \quad w_T = \frac{w_x}{J}; \quad s_T = \frac{s_x}{J}; \quad V_T = J V_x \quad (3)$$

a finned cylinder T is regarded as set up in an air stream of velocity V_T having the same value of the Reynolds Number $\frac{\rho V D}{\mu}$ and $\frac{k_a}{\mu c_p}, \frac{t}{D}, \frac{w}{D}, \frac{s}{D}$ as the finned cylinder x .

Let q_T be the surface heat-transfer coefficient corresponding to the velocity V_T and the finned cylinder dimensions w_T , s_T , t_T , and D_T . An inspection of equation (1) shows that q_x is related to q_T by the expression

$$q_x = \frac{1}{J} q_T \quad (4)$$

Data are presented in this report for determining q_T for a variety of values of t_T , w_T , s_T , and V_T . (See fig. 11.) As it has already been shown that q_T is a function mainly of the velocity and fin space, it is only necessary to find V_T and s_T and to determine q_T from figure 9.

Correction for altitude.—It was shown in figure 10 that q varies with the 0.796 power of the velocity. The general dimensional equation (1) can be reduced to give this relationship between the values of q and V ; it takes the form of

$$q = \frac{c_p}{D} (\rho V D)^{0.796} \mu^{0.204} f_1 \left[\left(\frac{k_a}{\mu c_p} \right), \left(\frac{t}{D} \right), \left(\frac{w}{D} \right), \left(\frac{s}{D} \right) \right] \quad (5)$$

where f_1 is a general function of $\frac{k_a}{\mu c_p}$, $\frac{t}{D}$, $\frac{w}{D}$, and $\frac{s}{D}$.

The quantities μ , c_p , and k_a depend on the temperature of the air, however, for the range of temperatures encountered in an ordinary altitude change c_p , $\frac{k_a}{\mu c_p}$, and $\mu^{0.204}$ are practically constant, causing a reduction in q of only 3.5 percent for a variation of altitude from sea level to 25,000 feet.

Neglecting the effect of variation of μ , c_p , and k_a it is evident from the above equation that q is a function of the mass flow ρV , and that the value of q at any altitude for a given finned cylinder is equal to the sea level q for the velocity giving the same mass flow.

This velocity is given by

$$V_{(\text{sea level})} = \frac{\rho_{(\text{altitude})}}{0.0734} V_{(\text{altitude})} \quad (6)$$

where 0.0734 is the weight density of air in pounds per cubic feet at 29.92 inches of Hg and 80° F., for which conditions the data have been corrected.

Materials of construction of fins.—The value of q is a function of the air flow around the cylinder and depends upon the constants of the fins only insofar as they influence the air flow. It is evident, therefore, that the values of q presented in this report hold also for fins constructed from materials other than steel.

DEVELOPMENT OF HEAT-FLOW EQUATION

By equating the heat conveyed from the surface of a fin by the air stream to the heat transmitted to the surface by conduction through the fin, a differential equation may be obtained, the solution of which leads to an expression for calculating the heat dissipated from finned cylinders as a function of the surface heat-transfer coefficient, the fin dimensions, and the average temperature of the cylinder wall.

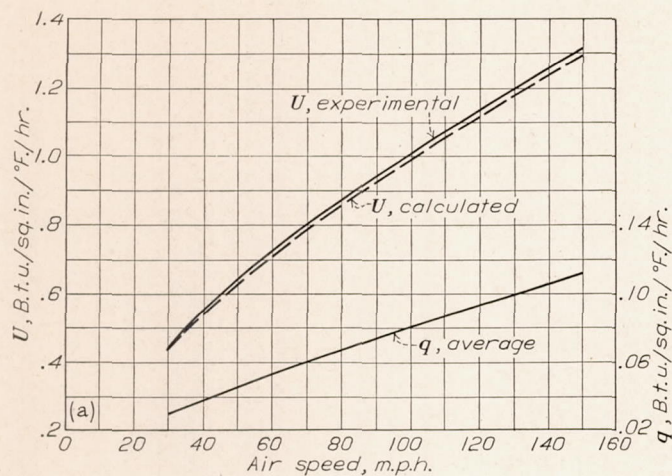
Harper and Brown (reference 2) show how the solution for the simple case of the straight rectangular fin can be corrected to apply for circular tapered fins by determining expressions for the correction for taper and for curvature. The expressions for these corrections are complicated and difficult to manipulate. In this section the general equation for the heat transmission through a circular tapered fin is given and, with the introduction of a number of assumptions, this equation is reduced to a simple equation similar to that for the straight rectangular fin. The solution of the simple equation is found to be adequate for the purpose of this report and, as will be shown later, checks experimental results closely.

The general equation for the flow of heat from a tapered fin (see fig. 12) derived by equating the heat entering the element of volume by conduction radially and circumferentially to the heat carried away from the exposed surfaces of the volume by convection, is

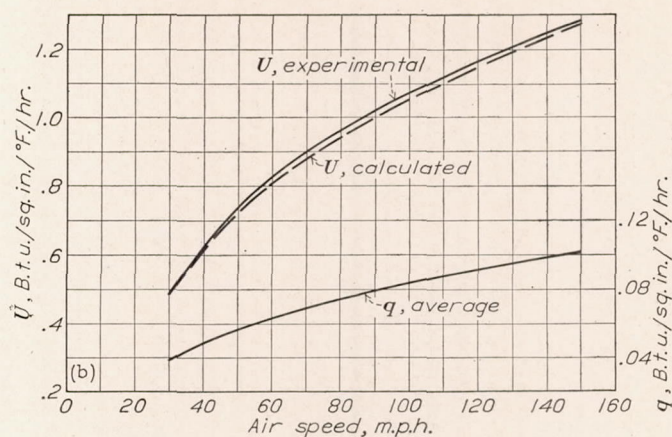
$$k \frac{\partial}{\partial R} \left(R t \frac{\partial \theta'}{\partial R} \right) + k \frac{t}{R} \frac{\partial^2 \theta'}{\partial \phi^2} = 2 q \frac{R \theta'}{\cos \alpha} \quad (7)$$

in which t is the variable thickness of fin, and q is the surface heat-transfer coefficient at any point. In equation (7) the assumption is made that the temperature does not vary through the thickness of the fin. Harper and Brown have shown that the error introduced by this assumption is negligible for a thin straight rectangular fin.

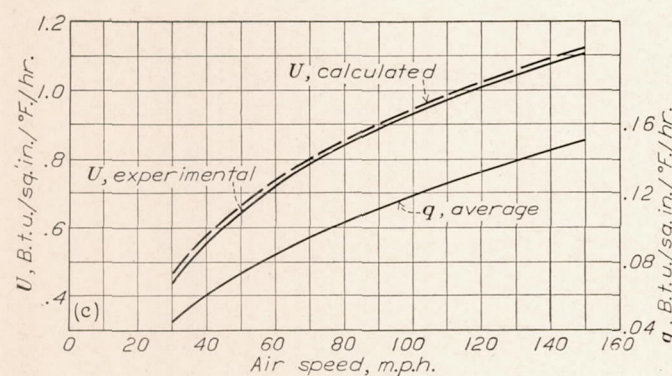
The values of q , t , and R in equation (7) vary from point to point on the fin and an exact solution of the



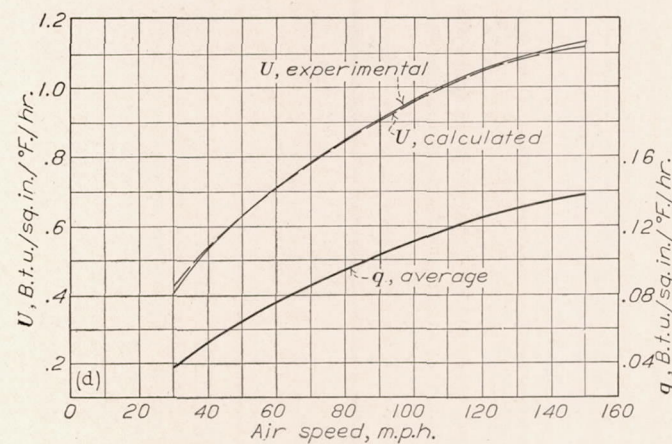
(a) Cylinder 0.15-0.97-0.075.



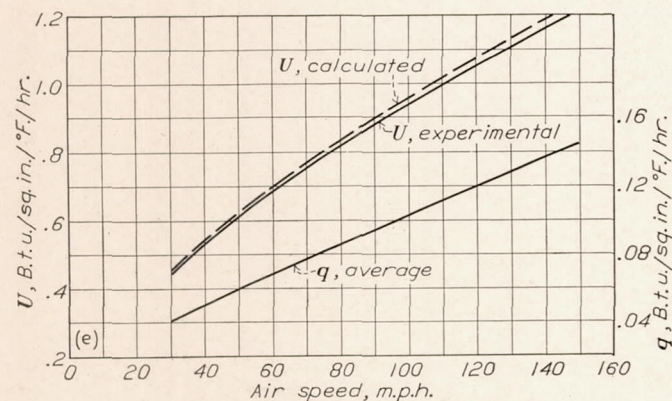
(b) Cylinder 0.20-1.22-0.10.



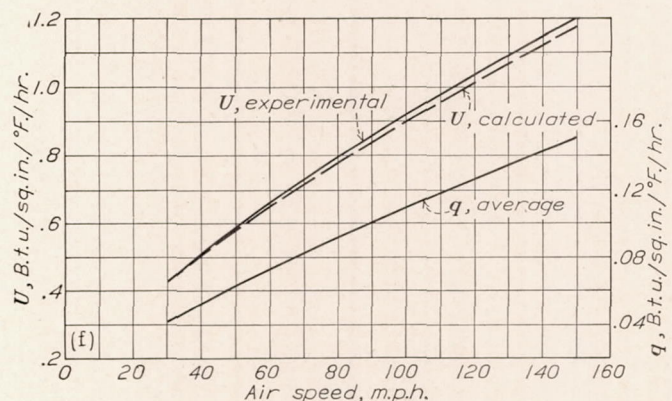
(c) Cylinder 0.25-1.22-0.065.



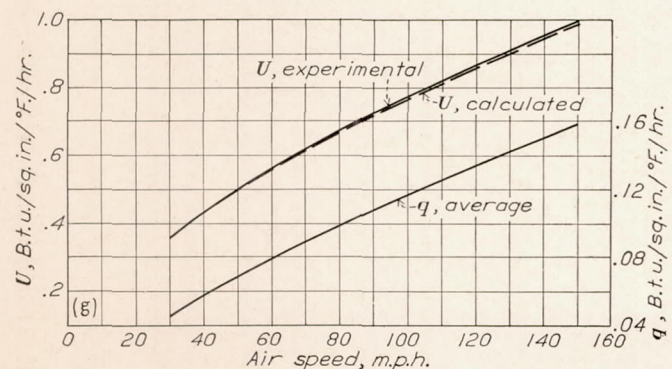
(d) Cylinder 0.25-1.22-0.09.



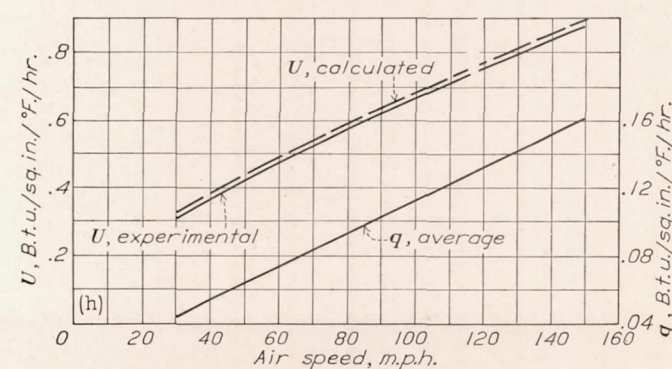
(e) Cylinder 0.25-1.22-0.115.



(f) Cylinder 0.30-1.32-0.13.

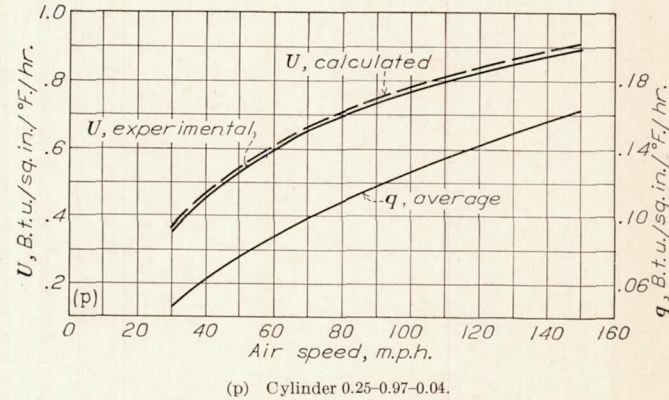
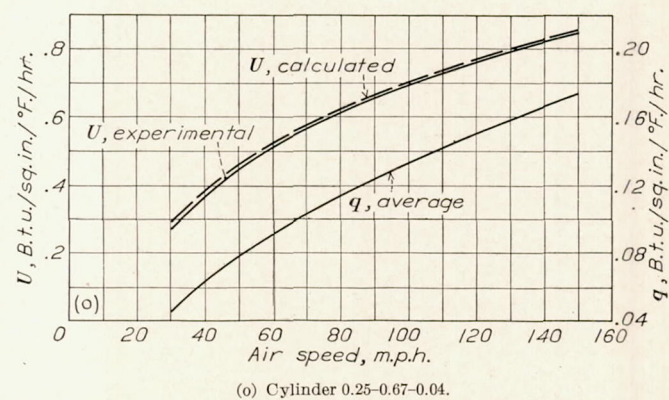
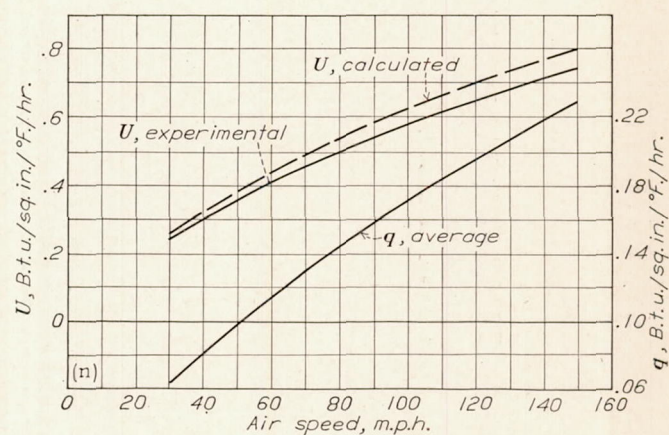
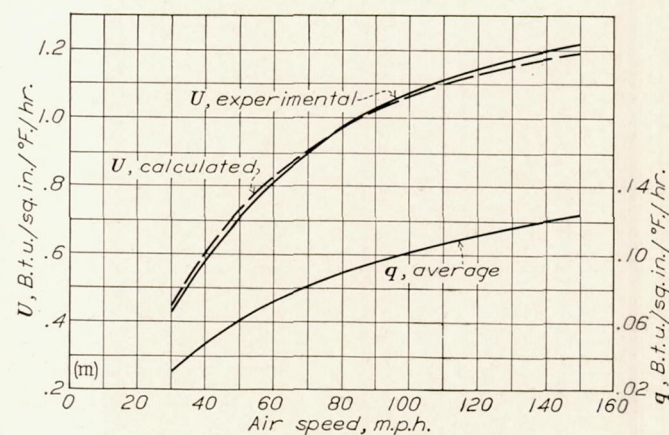
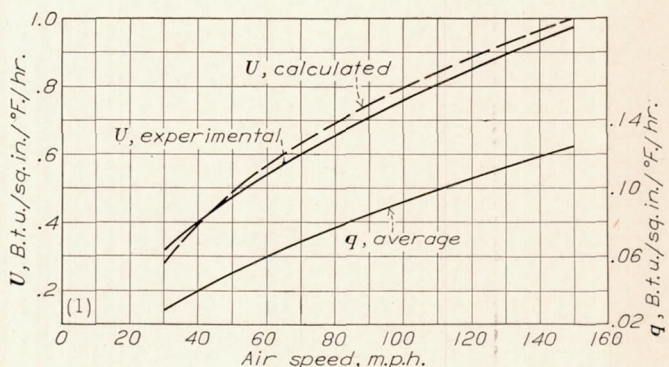
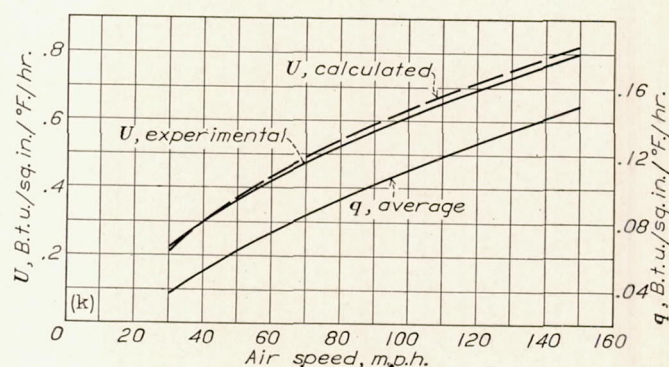
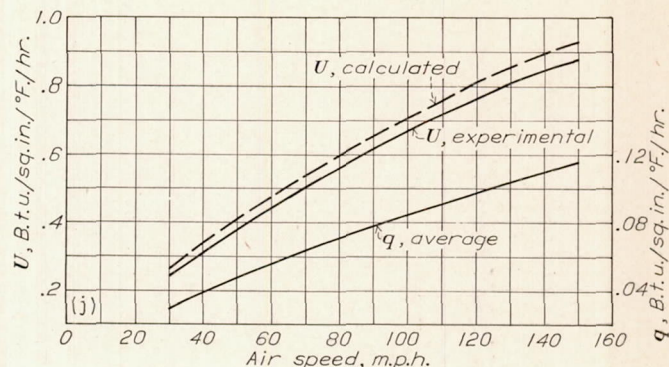
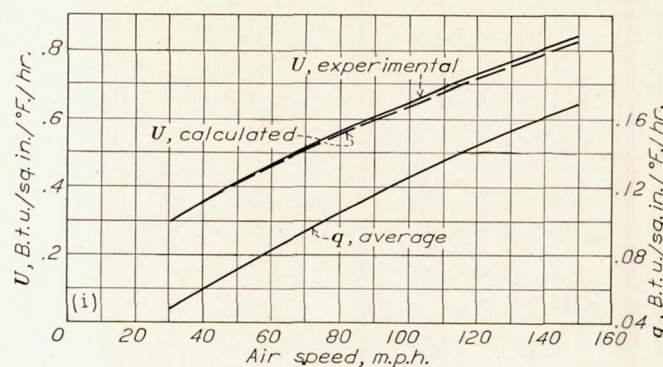


(g) Cylinder 0.40-1.32-0.18.



(h) Cylinder 0.50-1.47-0.23.

FIGURE 11.—Variation of average q with air speed and comparison of calculated and experimental values of U .

FIGURE 11.—Variation of average q with air speed and comparison of calculated and experimental values of U .—Continued.

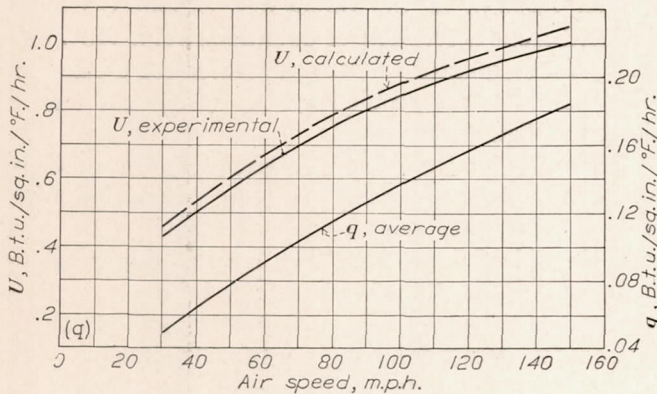
equation would lead to an expression too cumbersome for practical use. Considerable simplification can be effected by replacing R , t , and q by their average values. In general, the taper of the fins is so slight that $\cos \alpha$ can be taken as equal to 1. It will be shown later that the errors introduced by making these approximations are small.

Introducing these simplifications in equation (7) and integrating with respect to φ from 0 to π there results

$$ktR_a \frac{\partial^2}{\partial R^2} \left[\int_0^\pi \theta' d\varphi \right] + \frac{kt}{R_a} \frac{\partial \theta'}{\partial \varphi} \Big|_0^\pi = 2qR_a \int_0^\pi \theta' d\varphi \quad (8)$$

Since the temperature distribution around the fin is symmetrical with respect to the diameter parallel to the air stream,

$$\frac{\partial \theta'}{\partial \varphi} = 0 \text{ at } \varphi = \pi \text{ and } \varphi = 0.$$



(q) Cylinder 0.25-1.22-0.04.

FIGURE 11.—Variation of average q with air speed and comparison of calculated and experimental values of U .—Continued.

Let θ be the average temperature of the fin at any radius. Then by definition

$$\theta = \frac{1}{\pi} \int_0^\pi \theta' d\varphi$$

and equation (8) becomes

$$\frac{\partial^2 \theta}{\partial R^2} = a^2 \theta \quad (9)$$

where

$$a^2 = \frac{2q}{kt} \quad (10)$$

and t and q henceforth designate the average fin thickness and average surface heat-transfer coefficient, respectively. Equation (9) is similar to that for the straight rectangular fin and its solution is well known.

The heat dissipated from the fin tip can be accounted for as suggested by Harper and Brown by arbitrarily increasing the value of the width of the fin (w) by one-half the tip thickness (t_i).

Define

$$w' = w + \frac{1}{2} t_i \quad (11)$$

The solution of equation (9) is

$$\theta = \theta_b \frac{\cosh a(R - R_b - w')}{\cosh aw'} \quad (12)$$

subject to the conditions that

$$\theta = \theta_b \text{ when } R = R_b \text{ and } \frac{\partial \theta}{\partial R} = 0 \text{ when } R = R_b + w'$$

where θ_b is the average root temperature and R_b is the radius to the root.

The heat dissipated from the surface of a fin is, in general

$$H = 2 \int_{R_b}^{R_b + w'} \int_0^\pi 2q \frac{\theta' R dR d\varphi}{\cos \alpha}$$

Introducing the same simplifications as made in the derivation of equation (9) and replacing θ by the ex-

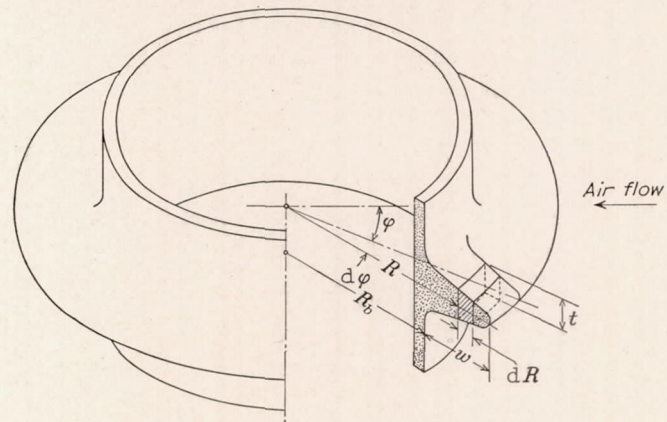


FIGURE 12.—Diagram of fin.

pression given in equation (12) the following expression results:

$$H = \frac{4\pi q \left(R_b + \frac{1}{2} w \right)}{a} \theta_b \tanh aw'$$

The amount of heat H_2 given off by that part of the cylinder wall exposed between two adjacent fins can be found approximately by assuming the wall surface heat-transfer coefficient to be the same as that of the fins.

$$H_2 = 2\pi R_b s_b q \theta_b$$

where s_b is the length of cylinder wall exposed between two adjacent fins.

The heat dissipated per square inch of cylinder-wall area per degree of cylinder-wall temperature is therefore

$$U = \frac{H_1 + H_2}{2\pi R_b (s + t) \theta_b} = \frac{q}{s + t} \left\{ \frac{2}{a} \left(1 + \frac{w}{2R_b} \right) \tanh aw' + s_b \right\} \quad (13)$$

It is now possible by means of equation (13) and the values of q given in the preceding section to calculate the heat dissipated from finned cylinders. Figure 11 shows the experimental values of U compared with the values calculated by means of equation (13) for various

air speeds and for the various cylinders tested. It will be noted that the agreement is very good for tapered fins as well as for rectangular fins.

OPTIMUM FINS

By means of equation (13) expressions can be obtained for determining the dimensions of the fins consistent with manufacturing practice that will dissipate the largest amount of heat for a given weight of material. It is evident that these same expressions also give the dimensions of the minimum-weight fins for a given heat dissipation.

For the purpose at hand, equation (13) can be simplified without introducing appreciable error by setting s_b equal to s and w' equal to w . The value of q can be replaced by a function of s

$$q = Ls^n$$

where $n = 0.322$ and L is a constant for a given velocity and density. (See fig. 9.) The volume of fin per unit of cylinder-wall area is given by

$$M = \frac{wt}{s+t} \left(1 + \frac{w}{2R_b} \right) \quad (14)$$

U in equation (13) may now be written as a function involving only s , t , and M as variables.

$$U = \frac{Ls^n}{s+t} \left[\frac{1}{\sqrt{\frac{2Ls^n}{kt}}} \left(1 + \sqrt{\frac{1+2M(s+t)}{R_bt}} \right) \right. \\ \left. \tanh R_b \left(\sqrt{1 + \frac{2M(s+t)}{R_bt}} - 1 \right) \sqrt{\frac{2Ls^n}{kt}} + s \right] \quad (15)$$

Obtaining the partial derivatives of U in equation (15) with respect to s and t respectively and setting each equal to zero the following relations result:

$$\frac{s + \left(1 + \frac{w}{R_b} \right) \left(s - n(s+t) \right)}{s + \left(1 + \frac{w}{R_b} \right) \left(s + n(s+t) \right)} \tanh a w - a w \operatorname{sech}^2 a w = \\ a s \left(\frac{1 + \frac{w}{R_b}}{1 + \frac{w}{2R_b}} \right) \frac{t + n(s+t)}{s + \left(1 + \frac{w}{R_b} \right) \left(s + n(s+t) \right)} \quad (16)$$

$$\frac{s - \left(1 + \frac{w}{R_b} \right) t}{s + \left(1 + \frac{w}{R_b} \right) (2s+t)} \tanh a w - a w \operatorname{sech}^2 a w = \\ a s \left(\frac{1 + \frac{w}{R_b}}{1 + \frac{w}{2R_b}} \right) \frac{t}{s + \left(1 + \frac{w}{R_b} \right) (2s+t)} \quad (17)$$

From a simultaneous solution of equations (14), (15), (16), and (17) the values of s , t , w , and U can be determined for a given value of M . The fins having these

dimensions will be the fins of maximum heat output for the volume of material M and will also be the minimum-weight fins that will dissipate the quantity of heat U . These calculations lead to fins which are much too thin and too closely spaced to be considered practical.

The problem is then to determine the dimensions of the optimum fins subject to the conditions that the fin thickness and space be not less than the values set as manufacturing limits. For a specified fin thickness t and a given value of M the required values of s , w , and U can be obtained from a solution of equations (16), (15), and (14). These quantities were calculated for a range of values of M and specified fin thickness t and are shown plotted in figure 13. The criterion of the weight economy $\left(\frac{U}{M} \right)$ was plotted in this figure instead of M . If s is specified instead of t , then for a given value of M the values of t , w , and U can be obtained from a solution of equations (17), (15), and (14). These quantities were calculated for a range of values of M and specified space, s , and are shown plotted in figure 14.

A simpler procedure for obtaining figure 13 is to substitute various pairs of values of s and t in equation (16), and for each pair of values solve for w by trial with the aid of figure 15. The corresponding values of M and U can then be determined from equations (14) and (15) respectively. A similar method can be applied for determining figure 14. The values of q used were obtained from figure 9 for an air speed of 76 miles per hour. The thermal conductivity used was that of steel (2.17 B.t.u./°F./in./hr.). The charts can be developed for other conditions by substituting the proper values of q and K in the above equations. The values of q can be determined for other air speeds and diameters by the methods presented in a previous section. The charts for any altitude will be the same as the sea-level chart for the velocity giving the same mass flow.

In practice, limits will be set for both the minimum allowable fin spacing and thickness. For these specifications in determining fins of minimum weight a result can be obtained from either figure 13 or 14 for a required value of U . Not more than one of these results will give dimensions sufficiently large to satisfy completely the specifications and be a solution of the problem.

In cases when neither chart gives a solution and only in such cases, the fins of minimum weight can be found by substituting the values of U and of the specified minimum thickness and spacing in equation (13) and solving for w .

An examination of figures 13 and 14 shows that the lower the specified minimum thickness and spacing the lighter are the fins that can be used to dissipate a given amount of heat. The curves also indicate that

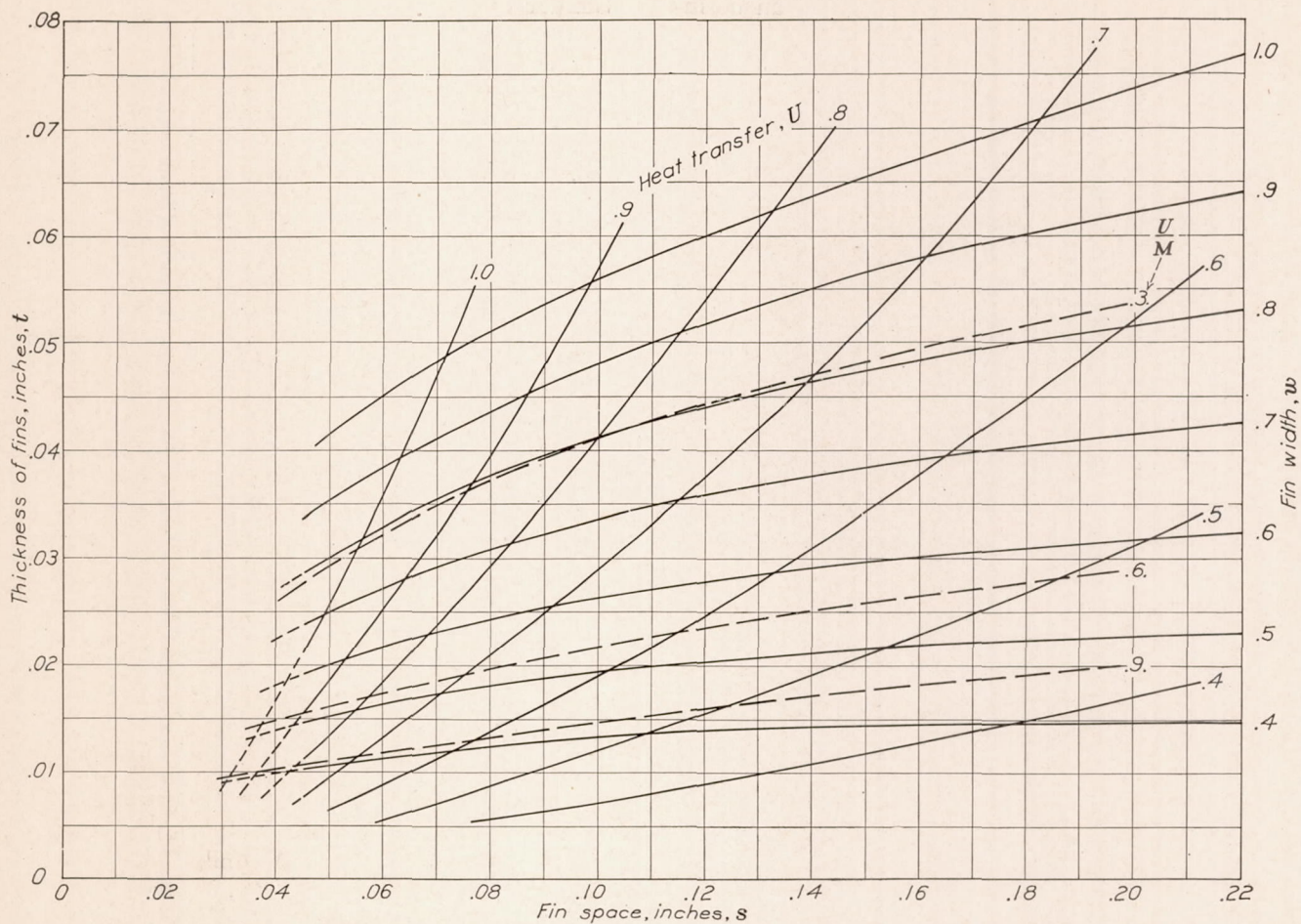


FIGURE 13.—Optimum fin dimensions for the condition in which the fin thickness is the specified governing dimension. ($V=76$ m.p.h., $D=4.66$ in., $k=2.17$ B.t.u./°F./in./hr.)

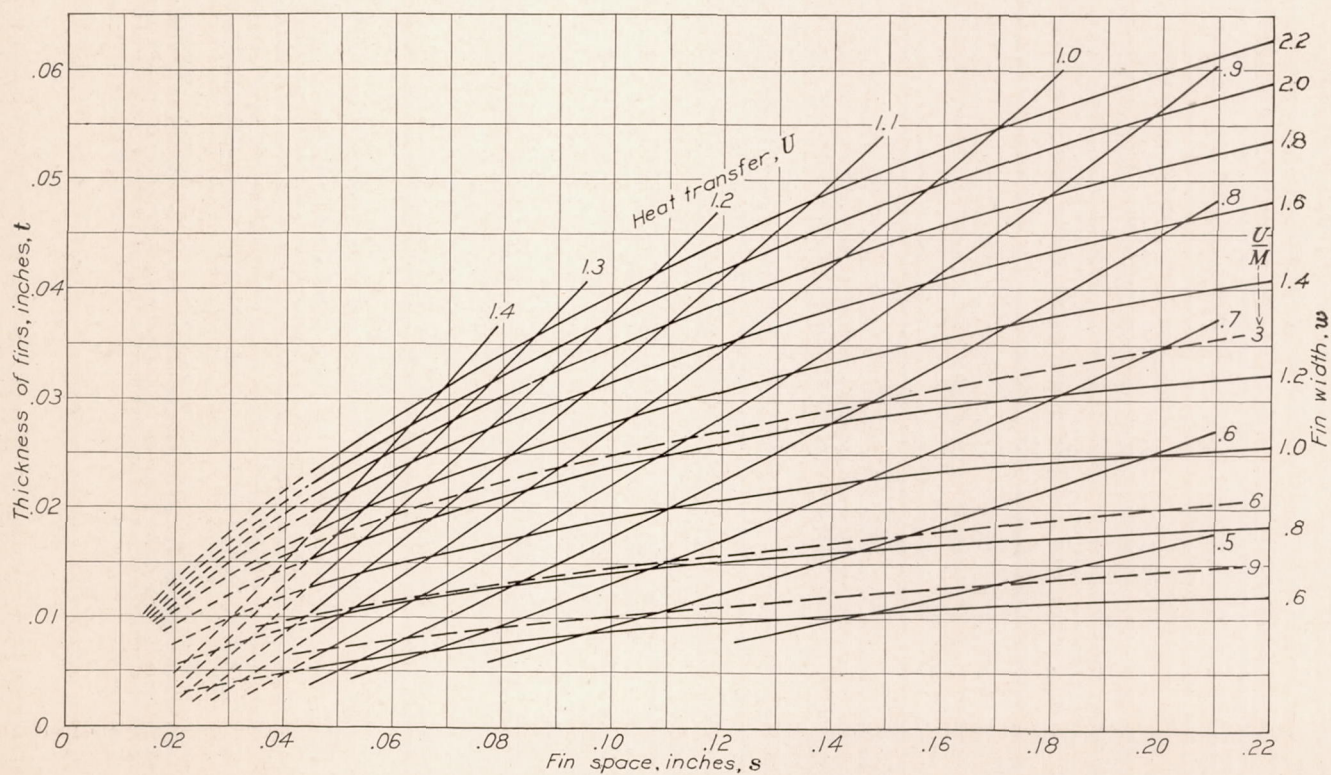


FIGURE 14.—Optimum fin dimensions for the condition in which the fin space is the specified governing dimension. ($V=76$ m.p.h., $D=4.66$ in., $k=2.17$ B.t.u./°F./in./hr.)

for a specified fin spacing the heat output can be increased by increasing the thickness and width, and that this gain is made at a sacrifice of weight economy, whereas for a specified thickness the heat output can be increased by decreasing the spacing and slightly increasing the width. For the latter case there is little loss in weight economy. Similar charts that have been plotted for other air speeds, cylinder diameters, and metals indicate the same trends.

The results of recently completed tests, now being prepared for publication, show that the equations presented here can also be applied to shrouded cylinders and to blower-cooled cylinders when the values of q for these cases are available.

APPLICATION OF RESULTS

Examples are here presented to illustrate the use of the equations and charts developed in this report.

Similar examples can be solved for cylinders provided with deflectors and for forced cooling when the values of q for these cases are available.

DETERMINATION OF HEAT TRANSFER FROM GIVEN FIN DESIGNS

Example 1: It is required to determine the heat dissipated from a 4.66-inch-diameter cylinder having an average cylinder-wall temperature of 320° F., when subjected to an air-stream velocity of 110 miles per hour at a sea-level pressure of 29.92 inches of Hg and an air temperature of 80° F. The cylinder is provided with tapered steel fins having the following dimensions:

Tip thickness (t_t)	0.020
Root thickness (t_b)	.040
Pitch (p)	.180
Width (w)	.700

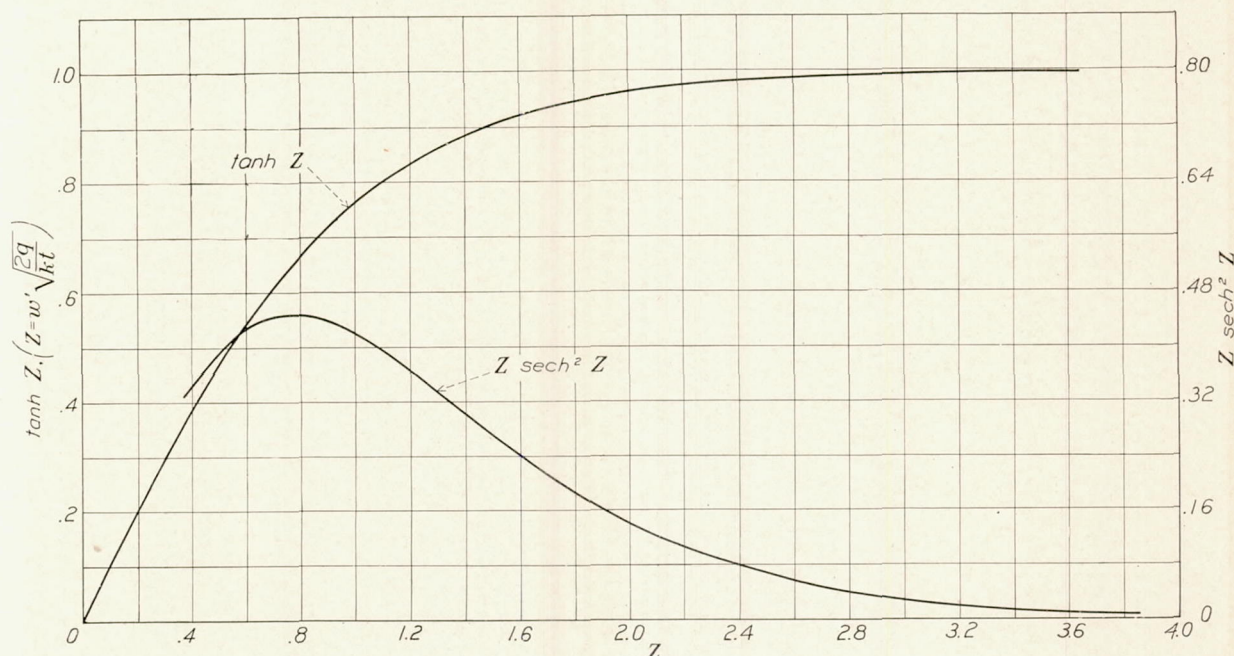


FIGURE 15.—Values of $\tanh Z$ and $Z \operatorname{sech}^2 Z$.

Two types of problems are solved for various conditions of air speed, cylinder diameter, metal conductivity, and altitude. The one type of example involves the determination of the heat transfer from given fin designs and the other involves the determination of the best fin design to meet given specifications.

The solution of several of the examples depends upon the use of the charts in figures 13 and 14 which were developed for a cylinder diameter of 4.66 inches, an air speed of 76 miles per hour, and a metal having a conductivity of 2.17 B.t.u./°F./in./hr. The solution of other examples of this type involving conditions other than these requires the construction of similar charts for the conditions desired. Additional charts covering a wide range of cylinder diameters and air speeds are being prepared at this laboratory.

$$U = \frac{q}{p} \left[\frac{2}{a} \left(1 + \frac{W}{2R_b} \right) \tanh a w' + s_b \right] \quad (\text{equation (13)})$$

$$\text{Average fin thickness } (t) = \frac{0.020 + 0.040}{2} = 0.03 \text{ inch.}$$

$$w' = w + \frac{1}{2} t_t = 0.70 + \frac{0.020}{2} = 0.710 \text{ inch} \quad (\text{equation (11)})$$

$$\text{Space between fins at root } (s_b) = p - t_b = 0.14 \text{ inch.}$$

For an air speed of 110 miles per hour and a fin space of 0.150 inch the value of the heat-transfer coefficient as read from figure 9 is 0.114 B.t.u./sq.in./°F./hr.

$$a = \sqrt{\frac{2q}{kt}} = \sqrt{\frac{2 \times 0.114}{2.17 \times 0.03}} = 1.87 \quad (\text{equation (10)})$$

$$U = \frac{0.114}{0.180} \left[\frac{2}{1.87} \left(1 + \frac{0.35}{2.33} \right) \tanh 1.87 \times 0.710 + 0.140 \right] \\ = 0.633 [1.23 \tanh 1.33 + 0.140]$$

From figure 15, $\tanh 1.33$ is 0.868.

$U = 0.765$ B.t.u. per square inch of cylinder-wall area per hour per $^{\circ}\text{F}$. temperature difference between the cylinder wall and cooling air.

The heat dissipated per square inch of cylinder-wall area is

$$Q = 0.765 (320 - 80) = 184 \text{ B.t.u. per hour}$$

Example 2: Let it be required to solve example 1 with aluminum-alloy fins replacing the steel fins and with all other conditions of the problem left unchanged.

Since q is independent of the conductivity of the metal, as previously pointed out, the only change introduced is the change in the value of the conductivity k in the equation for a .

For aluminum Y alloy, which is commonly used for cylinder heads, $k = 7.66$ B.t.u./ $^{\circ}\text{F}$./in./hr.

$$a = \sqrt{\frac{2 \times 0.114}{7.66 \times 0.03}} = 0.997 \quad (\text{equation (10)})$$

$$U = \frac{0.114}{0.18} \left[\frac{2}{0.997} \left(1 + \frac{0.35}{2.33} \right) \tanh 0.997 \times 0.710 + 0.140 \right] = 0.981$$

The heat dissipated per square inch of cylinder-wall area is

$$Q = 0.981 (320 - 80) = 235 \text{ B.t.u. per hour}$$

Example 3: Let it be required to solve example 1 for the case in which the cylinder is to be cooled at an air speed of 110 miles per hour at an altitude of 23,000 feet, all other conditions being left unchanged.

It was previously shown that the velocity at sea level giving the same mass flow as 110 miles per hour at 23,000 feet will give the same q for a given finned cylinder.

From the Standard Atmosphere Tables (reference 7) the weight density of air at 23,000 feet is 0.0368 pound per cubic foot and the temperature is -23°F .

The values of q plotted in figure 9 were corrected to a weight density of 0.0734 pound per cubic foot corresponding to 80°F . and 29.92 inches of Hg.

The required q can be found from figure 9 corresponding to a sea-level velocity of

$$V = \frac{0.0368}{0.0734} \times 110 = 55.1$$

miles per hour (equation (6)) and an average space between fins of 0.15 inch. The value is $q = 0.0665$.

The computations proceed along the lines of example 1

$$a = \sqrt{\frac{2 \times 0.0665}{2.17 \times 0.03}} = 1.43$$

$$U = \frac{0.0665}{0.180} \left\{ \frac{2}{1.43} \left(1 + \frac{0.35}{2.33} \right) \tanh 1.43 \times 0.710 + 0.140 \right\} = 0.507$$

The heat dissipated per square inch of cylinder-wall area is

$$Q = 0.507 (320 + 23) = 174 \text{ B.t.u. per hour}$$

Example 4: It is required to determine the heat dissipated from a 6-inch-diameter cylinder subject to the same conditions as the cylinder in example 1.

It was previously shown that in order to determine q from the data of the tested cylinder, for cylinders of diameter other than the one tested, it is necessary to apply the following method:

$$J = \frac{D}{4.66} = \frac{6}{4.66} = 1.287 \quad (\text{equation (2)})$$

$$JV = 110 \times 1.287 = 142 \text{ m.p.h.} \quad (\text{equation (3)})$$

$$\frac{s}{J} = \frac{0.150}{1.287} = 0.117 \text{ inch.}$$

From figure 9, the value of q_T corresponding to a velocity of 142 miles per hour and a space between fins of 0.117 inch is 0.130. The required value of q is

$$q = \frac{q_T}{J} = \frac{0.130}{1.287} = 0.101 \quad (\text{equation (4)})$$

The problem proceeds along the lines of example 1

$$a = \sqrt{\frac{2 \times 0.101}{2.17 \times 0.03}} = 1.76$$

$$U = \frac{0.101}{0.180} \left\{ \frac{2}{1.76} \left(1 + \frac{0.35}{3} \right) \tanh 1.76 \times 0.710 + 0.140 \right\} = 0.681$$

The heat dissipated per square inch of cylinder-wall area is

$$Q = 0.681 \times (320 - 80) = 164 \text{ B.t.u. per hour}$$

Example 5: To estimate the heat dissipated from the rear of a 6-inch-diameter aluminum cylinder having rectangular fins, when subjected to a velocity of the free air stream of 85 miles per hour:

Average fin thickness.....	inch..	0.07
Width.....	do....	.70
Average fin space.....	do....	.13
Temperature at rear of cylinder wall.....	$^{\circ}\text{F}$..	500
Air temperature.....	$^{\circ}\text{F}$..	80

$$J = \frac{6}{4.66} = 1.287 \quad (\text{equation (2)})$$

$$JV = 85 \times 1.287 = 109 \text{ m.p.h.} \quad (\text{equation (3)})$$

$$\frac{s}{J} = \frac{0.13}{1.287} = 0.101$$

The 0.20-1.22-0.10 cylinder (see fig. 1) has an average space of 0.10 inch, which is sufficiently close to the required value of 0.101. At the rear of the cylinder and at a velocity of 109 miles per hour the value of q_T is 0.096.

The desired value of q is

$$q = \frac{0.096}{1.287} = 0.0746 \quad (\text{equation (4)})$$

For aluminum, $k = 9.75$

$$a = \sqrt{\frac{2q}{kt}} = \sqrt{\frac{2 \times 0.0746}{9.75 \times 0.07}} = 0.468$$

$$p = 0.13 + 0.07 = 0.20$$

$$w' = 0.70 + 0.035 = 0.735 \text{ inch} \quad (\text{equation (11)})$$

$$U = \frac{0.0746}{0.20} \left\{ \frac{2}{0.468} \left(1 + \frac{0.35}{3} \right) \tanh 0.468 \times 0.735 + 0.13 \right\} = 0.641$$

The heat dissipated from the rear of the cylinder per square inch of cylinder-wall area is

$$Q = 0.641 (500 - 80) = 269 \text{ B.t.u. per hour}$$

DETERMINATION OF DIMENSIONS OF FINS OF MINIMUM WEIGHT

Example 6: It is desired to design steel fins of minimum weight to dissipate 154 B.t.u. per square inch per hour from a 4.66-inch-diameter cylinder subjected to an air-stream velocity of 76 miles per hour. The additional conditions are:

Average wall temperature.....	°F..	320
Air temperature.....	°F..	50
Average minimum specified fin thickness.....	inch..	0.02
Average minimum specified space.....	do.....	.06

$$U = \frac{Q}{\theta_b} = \frac{154}{320 - 50} = 0.570$$

For the specified conditions $s = 0.06$ and $U = 0.57$ it is found from figure 14 that t is less than 0.02 inch and therefore the result is not a solution.

Reference is now made to figure 13 which, for the specified $t = 0.02$ and $U = 0.57$, gives $s = 0.113$. This value of s is greater than the minimum specified s and is a solution. The remaining dimension is $w = 0.50$.

The desired fin construction has the dimensions

$$t = 0.02, s = 0.113, w = 0.50$$

With a tip radius of 0.005 inch and an average thickness of 0.02 inch, the base thickness must be 0.03 inch.

Example 7: As previously explained, there are some cases for which neither figure 13 nor figure 14 gives a solution. This example illustrates such a case.

It is desired to design steel fins of minimum weight to dissipate 200 B.t.u. per square inch per hour from a 4.66-inch-diameter cylinder in an air stream of 76 miles per hour.

Average wall temperature.....	°F..	310
Air temperature.....	°F..	90
Average minimum specified fin space.....	inch..	0.08
Average minimum specified fin thickness.....	do.....	.02

$$U = \frac{200}{310 - 90} = 0.91$$

For $U = 0.91$ and a specified thickness of 0.02 inch figure 13 gives $s = 0.051$. This value is less than the minimum specified fin space and is not a solution.

For $U = 0.91$ and a specified fin space of 0.08 inch figure 14 gives $t = 0.018$ inch and again no solution is reached.

For this case, as previously stated, recourse must be made to equation (15).

$$U = \frac{q}{t + s} \left\{ \frac{2}{a} \left(1 + \frac{w}{2R_b} \right) \tanh a w' + s_b \right\}$$

The value of q for a cylinder diameter of 4.66 inches, a velocity of 76 miles per hour and a fin space of 0.08 inch is (fig. 9) $q = 0.0695$.

$$a = \sqrt{\frac{2q}{kt}} = \sqrt{\frac{2 \times 0.0695}{2.17 \times 0.02}} = 1.79$$

Substituting $U = 0.91$, $s = 0.08$, $t = 0.02$

$$q = 0.0695 \text{ and } a = 1.79 \text{ in equation (15)}$$

$$0.91 = \frac{0.0695}{0.08 + 0.02} \left\{ \frac{2}{1.79} \left(1 + \frac{w' - 0.02}{2 \times 2.33} \right) \tanh 1.79 w' + 0.08 \right\}$$

$$\left\{ 0.998 + \frac{w'}{4.66} \right\} \tanh 1.79 w' = 1.100$$

The above equation can be solved by trial for w' and gives a value of $w' = 0.91$. The values for the hyperbolic tangent may be obtained from figure 15.

The required fin dimensions are,

$$w = 0.91 - \frac{0.02}{2} = 0.90 \text{ inch}$$

$$s = 0.08 \text{ inch}$$

$$t = 0.02 \text{ inch}$$

Example 8: It is desired to design steel fins of minimum weight to dissipate 200 B.t.u. per square inch per hour from a 4.66-inch-diameter cylinder in an air stream of 152 miles per hour at an altitude of 23,000 feet.

Average minimum specified fin thickness.....	inch..	0.02
Average minimum specified fin space.....	do.....	.06
Average wall temperature.....	°F..	300
Air temperature at 23,000 feet.....	°F..	-23

$$U = \frac{200}{300 + 23} = 0.619$$

Weight density of air at 23,000 feet is 0.0368 pound per cubic foot.

The velocity at a sea-level weight density of 0.0734 that will give the same mass flow and Reynolds Number and therefore the same q as 152 miles per hour at 23,000 feet altitude, is

$$V = 152 \times \frac{0.0368}{0.0734} = 76 \text{ miles per hour} \quad (\text{equation (6)})$$

The charts are correct for a weight density of 0.0734 pound per cubic foot and a speed of 76 miles per hour. When the equivalent velocity is different from 76 miles per hour other charts are required.

For $U=0.619$ and a specified thickness of 0.02-inch, figure 13 gives $s=0.097$. This value is greater than the minimum specified space of 0.06 and is a solution. The remaining fin dimension is $w=0.51$.

The required fin construction is

$$\begin{aligned} t &= 0.02 \text{ inch} \\ s &= 0.097 \text{ inch} \\ w &= 0.51 \text{ inch} \end{aligned}$$

Example 9: It is desired to design fins conforming to the conditions given in example 6 except that instead of the average cylinder-wall temperature, a maximum cylinder temperature of 500° F. is specified.

From measurements of the temperatures around the cylinder for air speeds from 30 to 150 miles per hour and for fin pitches varying from 0.10 to 0.30, the ratio of the maximum to the average cylinder-wall temperature difference was found to be given approximately by 1.35.

The average cylinder-wall temperature difference is then

$$\theta_b = \frac{500 - 80}{1.35} = 311^\circ \text{ F.}$$

It is now possible to proceed with the solution of the problem in a manner identical to that used in solving example 6.

CONCLUSIONS

Several conclusions may be enumerated:

1. The value of the surface heat-transfer coefficient varies mainly with the air velocity and the space between fins. The effect of the other fin dimensions is small.

2. The ratio of the maximum to the average cylinder-wall temperature difference for cylinders having fin pitches from 0.10 to 0.30 inch was found to be approximately 1.35.

3. The theoretical formula used in this report for calculating the heat dissipated from finned cylinders checks fairly closely the heat dissipation determined experimentally.

4. For the range of fins investigated the smaller the allowable thickness and space between fins, the more efficiently can the fins be designed from a weight consideration.

LANGLEY MEMORIAL AERONAUTICAL LABORATORY,
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
LANGLEY FIELD, VA., April 26, 1934.

SYMBOLS

Q , Over-all heat transfer, B.t.u. per square inch of cylinder-wall area per hour.

- U , Over-all heat-transfer coefficient, B.t.u. per square inch base area per hour, per °F. temperature difference between the cylinder wall and cooling air.
- q , Surface heat-transfer coefficient, B.t.u. per square inch total surface area per hour, per °F. temperature difference between the surface and the cooling air.
- θ , Average temperature difference between the cooling surface at any radius and the cooling air, °F.
- θ' , Temperature difference between any point on the cooling surface and the cooling air, °F.
- θ_b , Average temperature difference between the root of the fin and the cooling air, °F.
- k , Thermal conductivity of metal, B.t.u. per square inch per °F. through 1 inch per hour.
- k_a , Thermal conductivity of air, B.t.u. per foot per °F. per second.
- c_p , Specific heat of air at constant pressure, B.t.u. per pound per °F.
- μ , Viscosity of air (lb. sec.⁻¹ ft.⁻¹).
- ρ , Weight density of air, pound per cubic foot.
- V , Velocity of free air stream.
- D , Outer cylinder-wall diameter, inches.
- R_b , Radius from center of cylinder to fin root, inches ($R_b = D/2$).
- R_a , Average radius from center of cylinder to finned surface, inches.
- R , Radius from center of cylinder to any point on the fin.
- α , Angle between one surface of the fin and the median plane, degrees.
- φ , Angular position around the cylinder from front, degrees.
- t , Average fin thickness, inches.
- t_t , Fin-tip thickness, inches.
- t_b , Fin-root thickness, inches.
- s , Average distance through air space between adjacent fin surfaces, inches.
- s_b , Distance between adjacent fin surfaces at the fin root, inches.
- p , Fin pitch, inches ($p = s + t$).
- w , Fin width, inches.
- w' , Effective fin width ($w' = w + \frac{1}{2}t_t$).
- M , Volume of fins per square inch of cylinder-wall area.

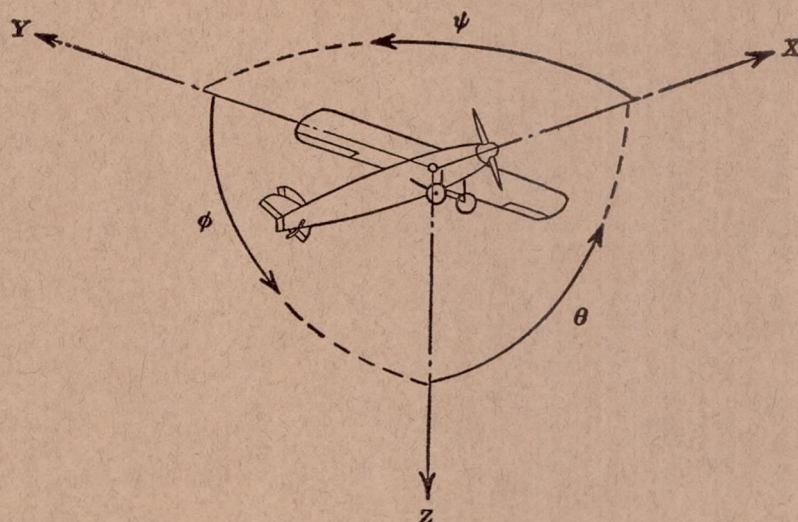
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Positive directions of axes and angles (forces and moments) are shown by arrows

Axis		Force (parallel to axis) symbol	Moment about axis			Angle		Velocities	
Designation	Sym- bol		Designation	Sym- bol	Positive direction	Designa- tion	Sym- bol	Linear (compo- nent along axis)	Angular
Longitudinal	X	X	Rolling	L	Y → Z	Roll	ϕ	u	p
Lateral	Y	Y	Pitching	M	Z → X	Pitch	θ	v	q
Normal	Z	Z	Yawing	N	X → Y	Yaw	ψ	w	r

Absolute coefficients of moment

$$C_l = \frac{L}{qbS}$$

(rolling)

$$C_m = \frac{M}{qcS}$$

(pitching)

$$C_n = \frac{N}{qbS}$$

(yawing)

Angle of set of control surface (relative to neutral position), δ . (Indicate surface by proper subscript.)

4. PROPELLER SYMBOLS

D , Diameter

p , Geometric pitch

p/D , Pitch ratio

V , Inflow velocity

V_s , Slipstream velocity

T , Thrust, absolute coefficient $C_T = \frac{T}{\rho n^2 D^4}$

Q , Torque, absolute coefficient $C_Q = \frac{Q}{\rho n^2 D^5}$

P , Power, absolute coefficient $C_P = \frac{P}{\rho n^3 D^5}$

C_s , Speed-power coefficient $= \sqrt[5]{\frac{\rho V^5}{P n^2}}$

η , Efficiency

n , Revolutions per second, r.p.s.

Φ , Effective helix angle $= \tan^{-1} \left(\frac{V}{2\pi r n} \right)$

5. NUMERICAL RELATIONS

1 hp. = 76.04 kg-m/s = 550 ft-lb./sec.

1 metric horsepower = 1.0132 hp.

1 m.p.h. = 0.4470 m.p.s.

1 m.p.s. = 2.2369 m.p.h.

1 lb. = 0.4536 kg.

1 kg = 2.2046 lb.

1 mi. = 1,609.35 m = 5,280 ft.

1 m = 3.2808 ft.